A-level

FURTHER MATHEMATICS
7367/3S

Paper 3 Statistics

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

226A7367/3S/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	21.25
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$\frac{7}{8}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	Uses $\int \lambda e^{-\lambda t}\ dt$ Condone missing dt or using x for t	1.1a	M1	$F(x) = \int_0^x \lambda e^{-\lambda t}\ dt$ $= \left[-e^{-\lambda t}\right]_0^x$ $= 1 - e^{-\lambda x}$
	Obtains correct integrated function May be unsimplified	1.1b	A1	
	Completes reasoned argument by substituting in limits and subtracting correct way round to show that $F(x) = 1 - e^{-\lambda x}$ or by solving $F(0) = 0$ to find the constant of integration Condone missing dt or using x for t in their solution but no other errors must be seen	2.1	R1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Obtains $F(1)$ = **AWRT** 0.865 or selects correct integral **PI** by correct final answer	1.1a	M1	$P(X > 1) = 1 - F(1)$ $= e^{-2\times 1}$ $= 0.135$
	Obtains correct value of $P(X > 1)$ **AWRT** 0.135	1.1b	A1	
	Total		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Translate situation into correct Poisson model **PI**	3.3	M1	$X \sim \text{Po}(26)$ $\text{P}(X \geq 30) = 1 - \text{P}(X \leq 29)$ $= 1 - 0.759$ $= 0.241$
	Uses their Poisson model to calculate $\text{P}(X \geq 30)$, $\text{P}(X > 30)$, $\text{P}(X \leq 29)$ or $\text{P}(X \leq 30)$	3.4	M1	
	Obtains the correct value of $\text{P}(X \geq 30)$ **AWRT** 0.241	1.1b	A1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Compares either their 26 with 100 or their $\sqrt{26}$ with 10	3.5b	M1	The Poisson model is not valid Variance of total of daisies and dandelions, 10^2, is not approximately equal to the mean, 26
	Concludes that because the mean/variance of the model is not approximately equal to 100 or because the standard deviation of the model is not approximately equal to 10, the Poisson model is not valid Must not use or imply $\neq$	2.4	A1F	
	Total		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains the correct value of s^2 **AWRT** 28.45 or 28.455 given to three decimal places or s **AWRT** 5.334 **PI** by correct calculation seen within their formula for the confidence interval	1.1b	B1	$\bar{x} = 86.5$ $s^2 = 28.45$ $t_{11} = 3.106$ $86.5 \pm 3.106 \times \sqrt{\frac{28.45}{12}}$ Therefore, the confidence interval is (81.7, 91.3)
	Obtains correct t_{11} value **AWRT** 3.106	1.1b	B1	
	Uses the correct formula for the confidence interval using the correct value or calculation of $\bar{x}$ and their s^2 and t values Condone use of **AWRT** 2.58 instead of a t value	1.1a	M1	
	Completes reasoned argument by substituting the correct values into the correct formula to show that the confidence interval is (81.7, 91.3)	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Infers null hypothesis is accepted as 85 lies within the confidence interval Must see 85 or reference to the proposed population mean	2.2b	E1	The null hypothesis is accepted as 85 lies within the confidence interval
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Concludes in context. Must refer to the mean mass of apples (Conclusion must not be definite)	3.2a	E1	Insufficient evidence to suggest that the mean mass of apples is different from 85 grams
	Total		**1**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Translate situation into at least one correct equation involving probabilities $a + b + c = 1$ or $b + 2c = 1.2$ or $b + 4c - 1.2^2 = 0.56$ **OE** $a + b + c = 1$ may be implied by finding values of a, b and c which add to 1	3.1a	B1	$a + b + c = 1$ $b + 2c = 1.2$ $b + 4c - 1.2^2 = 0.56 \Rightarrow b + 4c = 2$ $b = 0.4$ $c = 0.4$ $a = 0.2$
	Obtains at least two correct equations involving probabilities $a + b + c = 1$ may be implied by finding values of a, b and c which add to 1	1.1b	B1	
	Obtains three correct equations involving probabilities $a + b + c = 1$ may be implied by finding values of a, b and c which add to 1	1.1b	B1	
	Attempts to solve their simultaneous equations by attempting to find a value for one of a, b or c	1.1a	M1	
	Obtains correct value of one of a, b or c	1.1b	A1	
	Deduces that $a = 0.2$, $b = 0.4$ and $c = 0.4$	2.2a	A1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Use the formula $\text{Var}(X - 2Y - 11)$ $= \text{Var}(X) + 2^2\text{Var}(Y)$ to obtain $0.56 + 2^2 \times 15$ Condone one slip leading to either using $\text{Var}(X) + 2\text{Var}(Y)$ or $\text{Var}(X) - 2^2\text{Var}(Y)$	1.1a	M1	$\text{Var}(X - 2Y - 11)$ $= \text{Var}(X) + 2^2\text{Var}(Y)$ $= 0.56 + 2^2 \times 15$ $= 60.56$
	Obtains the correct value of $\text{Var}(X - 2Y - 11)$	1.1b	A1	
	Total		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	States both hypotheses using correct language Variables need to be stated in at least the null hypothesis	2.5	B1	H_0: There is no association between country and air quality H_1: There is an association between country and air quality
	Translate situation into expected contingency table for χ^2 model **PI**	3.3	M1	*Expected* | 1 | 2 *A* | 95.504 | 92.496 *B* | 158.496 | 153.504
	Uses χ^2 model to calculate test statistic **PI** Condone missing modulus sign Condone use of $\sum\frac{(O-E)^2}{E}$	3.4	M1	$\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}=$ $\frac{(\lvert 87-95.504\rvert-0.5)^2}{95.504}+\frac{(\lvert 167-158.496\rvert-0.5)^2}{158.496}$ + $\frac{(\lvert 101-92.496\rvert-0.5)^2}{92.496}+\frac{(\lvert 145-153.504\rvert-0.5)^2}{153.504}$ = 2.1849 χ^2 cv for 1 df = 2.706
	Obtains the correct value of $\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}$ **AWRT** 2.2	1.1b	A1	2.1849 < 2.706 Accept H_0 No evidence to suggest/support that there is an association between country and air quality
	Obtains the correct critical value for the test **AWRT** 2.7 or corresponding probability of test statistic **AWRT** 0.14	1.1b	B1	
	Evaluates χ^2 – test statistic by correctly comparing their critical value with their test statistic or their probability with 0.1	3.5a	R1	
	Infers H_0 accepted **FT** their calculated value of $\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}$	2.2b	E1F	

	Concludes in context (Conclusion must not be definite) **FT** their incorrect rejection of H_0 provided that it is consistent with their comparison	3.2a	E1F	
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Interprets Type I error in context	3.2a	E1	To conclude that there is an association between country and air quality when there is not
	Total		**1**	

	Question total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Forms a correct equation	1.1a	M1	$e^{5k} - 1 = 1$
	Completes reasoned argument to obtain the correct exact value of $k = \frac{1}{5}\ln 2$ The typical solution shows the minimum evidence required	2.1	R1	$e^{5k} = 2$ $5k = \ln 2$ $k = \frac{1}{5}\ln 2$
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Forms a correct equation	1.1a	M1	$e^{\frac{m}{5}\ln 2} - 1 = 0.5$
	Uses logarithms to correctly rearrange their equation into the form $pm = q$ where p and q are constants and m is their median	1.1a	M1	$e^{\frac{m}{5}\ln 2} = 1.5$ $\frac{m}{5}\ln 2 = \ln 1.5$
	Completes reasoned argument to obtain the correct exact value of the median $= 5\frac{\ln 3}{\ln 2} - 5$ The typical solution shows the minimum evidence required $\frac{1}{5}\ln 2$ may be left as k until the final line	2.1	R1	$\frac{m}{5}\ln 2 = \ln 3 - \ln 2$ $m = 5\frac{\ln 3}{\ln 2} - 5$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Differentiates F(x) to find the probability density function and obtains a function of the form Ae^{kx} where A is a non-zero constant	3.1a	M1	$\mathrm{f}(x) = \left(\frac{1}{5}\ln 2\right)e^{\frac{x}{5}\ln 2}$ E(X) = $\frac{1}{5}\ln 2\int_0^5 x\mathrm{e}^{\frac{x}{5}\ln 2}\,\mathrm{d}x$
	Forms an integral of the form $A\int x\mathrm{e}^{kx}\,\mathrm{d}x$ where A is a non-zero constant Condone missing $\mathrm{d}x$	1.1a	M1	$= \left[x\mathrm{e}^{\frac{x}{5}\ln 2}\right]_0^5 - \int_0^5 \mathrm{e}^{\frac{x}{5}\ln 2}\,\mathrm{d}x$ $= \left[x\mathrm{e}^{\frac{x}{5}\ln 2} - \frac{5}{\ln 2}\mathrm{e}^{\frac{x}{5}\ln 2}\right]_0^5$
	Uses integration by parts the correct way round to rearrange integral Condone one slip provided intent is clear	3.1a	M1	$= \left(10 - \frac{10}{\ln 2}\right) - \left(-\frac{5}{\ln 2}\right)$ $= 10 - \frac{5}{\ln 2}$
	Obtains the correct integrated function, possibly in terms of k and x Components of the integrated function may be seen on different lines	1.1b	A1	
	Substitutes limits of 0 and 5 and subtracts the correct way round to a changed function from their $Axe^{\frac{x}{5}\ln 2}$	1.1a	M1	
	Completes reasoned argument to find E(X) = $10 - \frac{5}{\ln 2}$ $\frac{1}{5}\ln 2$ may be left as k until the final line	2.1	R1	
	Total		**6**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Uses rectangular distribution model to obtain the correct value of P(X > 10.5)	3.4	B1	P(X > 10.5) = $\frac{1}{4} \times 1.5 = 0.375$ 0.375 < 0.4 Lianne will not buy the battery
	Correctly compares their value of P(X > 10.5) with 0.4 and interprets the result in context	3.2a	E1F	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Recognises limitation of rectangular distribution in modelling situation with reference to the shape of the histogram	3.5b	E1	The frequency density on the histogram is not approximately level between 8 and 12 hours Use the normal distribution instead
	Refines the model by suggesting the use of the normal distribution	3.5c	B1	
	Total		**2**	

	Question total		**4**	

	Paper total		**50**	