AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level
FURTHER MATHEMATICS

Paper 2

Thursday 6 June 2019 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Given that z is a complex number, and that z^* is the complex conjugate of z, which of the following statements is **not** always true?

Circle your answer.

[1 mark]

$(z^*)^* = z$ $\quad$ $zz^* = |z|^2$ $\quad$ $(-z)^* = -(z^*)$ $\quad$ $z - z^* = z^* - z$

2 Which of the straight lines given below is an asymptote to the curve

$$y = \frac{ax^2}{x - 1}$$

where a is a non-zero constant?

Circle your answer.

[1 mark]

$y = ax + a$ $\quad$ $y = ax$ $\quad$ $y = ax - a$ $\quad$ $y = a$

3 The set $\mathcal{A}$ is defined by $\mathcal{A} = \{x : -\sqrt{2} < x < 0\} \cup \{x : 0 < x < \sqrt{2}\}$

Which of the inequalities given below has $\mathcal{A}$ as its solution?

Circle your answer.

[1 mark]

$|x^2 - 1| > 1$ $\quad$ $|x^2 - 1| \geq 1$ $\quad$ $|x^2 - 1| < 1$ $\quad$ $|x^2 - 1| \leq 1$

4 The positive integer k is such that

$$\sum_{r=1}^{k}(3r-k)=90$$

Find the value of k.

[3 marks]

Turn over for the next question

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5 A curve has equation $y = \cosh x$

Show that the arc length of the curve from $x = a$ to $x = b$, where $0 < a < b$, is equal to

$$\sinh b - \sinh a$$

[4 marks]

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6 A circle C in the complex plane has equation $|z - 2 - 5\mathrm{i}| = a$

The point z_1 on C has the least argument of any point on C, and $\arg(z_1) = \frac{\pi}{4}$

Prove that $a = \frac{3\sqrt{2}}{2}$

[6 marks]

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7 The points A, B and C have coordinates $A(4, 5, 2)$, $B(-3, 2, -4)$ and $C(2, 6, 1)$

7 (a) Use a vector product to show that the area of triangle ABC is $\frac{5\sqrt{11}}{2}$

[4 marks]

7 (b) The points A, B and C lie in a plane.

Find a vector equation of the plane in the form $\mathbf{r}.\mathbf{n} = k$

[1 mark]

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7 (c) Hence find the exact distance of the plane from the origin.

[1 mark]

Turn over for the next question

8 A parabola P_1 has equation $y^2 = 4ax$ where $a > 0$

P_1 is translated by the vector $\begin{bmatrix} b \\ 0 \end{bmatrix}$, where $b > 0$, to give the parabola P_2

8 (a) The line $y = mx$ is a tangent to P_2

Prove that $m = \pm\sqrt{\frac{a}{b}}$

Solutions using differentiation will be given no marks.

[4 marks]

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8 (b) The line $y = \sqrt{\frac{a}{b}}\,x$ meets P_2 at the point *D*.

The finite region *R* is bounded by the x-axis, P_2 and a line through *D* perpendicular to the x-axis.

The region *R* is rotated through 2π radians about the x-axis to form a solid.

Find, in terms of a and b, the volume of this solid.

Fully justify your answer.

[5 marks]

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9 (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{M} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{-3}{5} & \frac{13}{10} \end{bmatrix}$$

[5 marks]

9 (b) Find matrices **U** and **D** such that **D** is a diagonal matrix and $\mathbf{M} = \mathbf{UDU}^{-1}$

[2 marks]

9 (c) Given that $\mathbf{M}^n \rightarrow \mathbf{L}$ as $n \rightarrow \infty$, find the matrix **L**.

[4 marks]

9 (d) The transformation represented by **L** maps all points onto a line.

Find the equation of this line.

[2 marks]

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10 Prove by induction that $f(n) = n^3 + 3n^2 + 8n$ is divisible by 6 for all integers $n \geq 1$

[7 marks]

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11 The line L_1 has equation

$$\frac{x-2}{3}=\frac{y+4}{8}=\frac{4z-5}{5}$$

The line L_2 has equation

$$\left(\mathbf{r}-\begin{bmatrix}-2\\0\\3\end{bmatrix}\right)\times\begin{bmatrix}2\\1\\3\end{bmatrix}=\mathbf{0}$$

Find the shortest distance between the two lines, giving your answer to three significant figures.

[8 marks]

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12 Abel and Bonnie are trying to solve this mathematical problem:

$z = 2 - 3\mathrm{i}$ is a root of the equation
$2z^3 + mz^2 + pz + 91 = 0$

Find the value of m and the value of p.

Abel says he has solved the problem.

Bonnie says there is not enough information to solve the problem.

12 (a) Abel's solution begins as follows:

Since $z = 2 - 3\mathrm{i}$ is a root of the equation,
$z = 2 + 3\mathrm{i}$ is another root.

State **one extra** piece of information about m and p which could be added to the problem to make the beginning of Abel's solution correct.

[1 mark]

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12 (b) Prove that Bonnie is right.

[4 marks]

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13 (a) Explain why $\int_3^\infty x^2 e^{-2x}\,dx$ is an improper integral.

[1 mark]

13 (b) Evaluate $\int_3^\infty x^2 e^{-2x}\,dx$

Show the limiting process.

[9 marks]

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14 Let

$$S_n = \sum_{r=1}^{n} \frac{1}{(r+1)(r+3)}$$

where $n \geq 1$

14 (a) Use the method of differences to show that

$$S_n = \frac{5n^2 + an}{12(n+b)(n+c)}$$

where a, b and c are integers.

[6 marks]

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14 (b) Show that, for any number k greater than $\frac{12}{5}$, if the difference between $\frac{5}{12}$ and S_n is less than $\frac{1}{k}$, then

$$n > \frac{k - 5 + \sqrt{k^2 + 1}}{2}$$

[6 marks]

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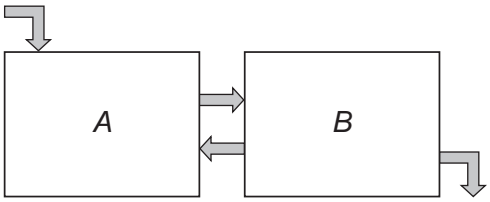

Two tanks, *A* and *B*, each have a capacity of 800 litres.

At time $t = 0$ both tanks are full of pure water.

When $t > 0$, water flows in the following ways:

- Water with a salt concentration of μ grams per litre flows into tank *A* at a constant rate
- Water flows from tank *A* to tank *B* at a rate of 16 litres per minute
- Water flows from tank *B* to tank *A* at a rate of r litres per minute
- Water flows out of tank *B* through a waste pipe
- The amount of water in each tank remains at 800 litres.

At time t minutes $(t \geq 0)$ there are x grams of salt in tank *A* and y grams of salt in tank *B*.

This system is represented by the coupled differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 36 - 0.02x + 0.005y \qquad (1)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 0.02x - 0.02y \qquad (2)$$

15 (a) Find the value of r.

[2 marks]

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15 (b) Show that $\mu = 3$

[3 marks]

15 (c) Solve the coupled differential equations to find both x and y in terms of t.

[9 marks]

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