Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/4C

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1	Correct method to find an equation in $\bar{x}$	M1	1.1b
	$-3\times2+4\times3+2\times p=9\bar{x}$ $\quad(6+2p=9\bar{x})$	A1	1.1b
	Correct method to find an equation in $\bar{y}$	M1	1.1b
	$3\times2+4\times1+2\times p=9\bar{y}$ $\quad 10+2p=9\bar{y}$	A1	1.1b
	$(9\bar{x})^2+(9\bar{y})^2=(6+2p)^2+(10+2p)^2$ $\left(=136+64p+8p^2\right)$	M1	1.1b
	$=8\left[(p+4)^2+17-16\right]$	M1	3.1a
	$\Rightarrow p=-4$	A1	2.2a
			(7 marks)

Notes:

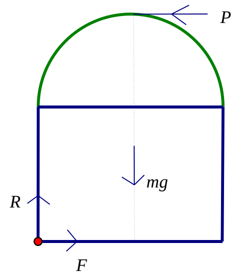
	M1	Take moments about axis parallel to $x=0$. Need all terms and dimensionally correct.
	A1	Correct unsimplified equation in $\bar{x}$. Seen or implied
	M1	Take moments about axis parallel to $y=0$. Need all terms and dimensionally correct.
	A1	Correct unsimplified equation in $\bar{y}$. Seen or implied
	M1	Use of Pythagoras to find distance (or square of distance) from origin
	M1	Correct strategy to find value of p to minimise the distance e.g. use of calculus or complete the square
	A1	Correct answer only

Question	Scheme	Marks	AOs
2(a)	$2.4\bar{y} = \frac{1}{2}\int y^2 \mathrm{d}x = \frac{1}{2}\int \left\{64\mathrm{e}^{-2x}\right\} \mathrm{d}x$	M1	2.1
	$= -16\left[\mathrm{e}^{-2x}\right]_{\ln 2}^{\ln 5}$	A1	1.1b
	Complete strategy to find $\bar{y}$	M1	3.1a
	$2.4\bar{y} = -16\mathrm{e}^{-\ln 25} + 16\mathrm{e}^{-\ln 4} = \frac{16}{4} - \frac{16}{25} = \frac{84}{25}$, $\bar{y} = \frac{84}{25} \times \frac{10}{24} \left(= \frac{7}{5}\right) = 1.4$ *	A1*	2.2a
		(4)	
(b)	$2.4\bar{x} = \int \left(8x\mathrm{e}^{-x}\right) \mathrm{d}x$	M1	2.1
	$\left(= \left[-8x\mathrm{e}^{-x} - 8\mathrm{e}^{-x}\right]_{\ln 2}^{\ln 5}\right)$		
	$= -\frac{8}{5}(\ln 5 + 1) + \frac{8}{2}(\ln 2 + 1)(= 2.5974.....)$	M1	1.1b
	$\bar{x} = 1.08$	A1	1.1b
	Complete strategy to find θ	M1	3.1a
	$\tan\theta° = \frac{\ln 5 - \bar{x}}{8\mathrm{e}^{-\ln 5} - 1.4}(= 2.63......)$	A1ft	3.4
	$\theta = 69$	A1	1.1b
		(6)	
			(10 marks)

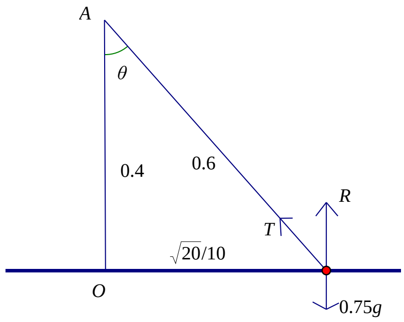
Notes:		
(a)	M1	Moments equation to obtain terms of the correct form (with or without limits) Allow if area (2.4) not seen
	A1	Correct unsimplified answer (with or without limits) Allow if area (2.4) not seen
	M1	Complete strategy for $\bar{y}$: use of moments equation with correct use of limits and division by area
	A1*	Use moments equation and given area to deduce given answer from correct working
(b)	M1	Use correct integral (with or without limits). Allow if area (2.4) not seen
	M1	Correct use of correct limits in an integral of the correct form and 2.4
	A1	Correct answer (1.0822….)
	M1	Complete strategy to find θ e.g find $\bar{x}$ and then use trig to find appropriate angle
	A1ft	Use the model to find a relevant angle. Follow their $\bar{x}$
	A1	2 s.f. or better 69.22...

Question	Scheme	Marks	AOs
3(a)	Form differential equation: $0.5a = 0.5v\frac{\mathrm{d}v}{\mathrm{d}x} = -kv^2$	M1	2.5
	$\Rightarrow \int \frac{1}{2v}\mathrm{d}v = \int -k\mathrm{d}x$	M1	2.1
	$\frac{1}{2}\ln v = -kx + C$	A1	1.1b
	$x = 1,\ v = 4 \quad \frac{1}{2}\ln 4 = -k + C$ $x = 2,\ v = 2 \quad \frac{1}{2}\ln 2 = -2k + C$	M1	3.1a
	$\Rightarrow k = \frac{1}{2}(\ln 4 - \ln 2) = \frac{1}{2}\ln 2\ ,\ \ C = \frac{1}{2}\ln 8\ :\ \ \ln v = -x\ln 2 + \ln 8$	A1	1.1b
	$\ln v = x\ln\frac{1}{2} + \ln 8,\ \ v = 8\times\left(\frac{1}{2}\right)^x$	A1	2.2a
	$\left(a = 8,\ \ b = \frac{1}{2}\right)$		
		(6)	
(b)	$0.5\frac{\mathrm{d}v}{\mathrm{d}t} = -kv^2$ (follow their k)	M1	2.5
	$\int \frac{1}{v^2}\mathrm{d}v = \int -\ln 2\mathrm{d}t \quad \Rightarrow -\frac{1}{v} + C' = -t\ln 2$	M1	2.1
	$\Rightarrow \left[-\frac{1}{v}\right]_4^2 = \left[-t\ln 2\right]_0^T$	M1	1.1b
	$-\frac{1}{2} + \frac{1}{4} = -T\ln 2\ ,\quad T = \frac{1}{4\ln 2}\quad *$	A1*	2.2a
		(4)	
(b) alt	$v = \frac{8}{2^x} = \frac{\mathrm{d}x}{\mathrm{d}t}$ (follow their v)	M1	2.5
	$\int 2^x\mathrm{d}x = \int 8\,\mathrm{d}t \quad \Rightarrow \frac{2^x}{\ln 2} = 8t + C'$	M1	2.1
	$\left[\frac{2^x}{\ln 2}\right]_1^2 = \left[8t\right]_0^T$	M1	1.1b
	$\frac{1}{\ln 2}(4-2) = 8T,\quad T = \frac{1}{4\ln 2}\quad *$	A1*	2.2a
		(4)	
			(10 marks)

Notes:		
(a)	M1	Form differential equation in *v* and *x*. Condone sign error
	M1	Separate and integrate to form equation in *v* and *x*. Condone missing constant of integration
	A1	Any equivalent form. Condone missing constant of integration.
	M1	Complete strategy to use the differential equation and boundary conditions to find *v*
	A1	Correct expression in *v* and *x* in any form. Accept $\ln v = -0.693..x + 2.079...$
	A1	Expression in the required form. Do not need to see a separate statement of the values of *a* and *b*.
		If mass is omitted from the differential equation can score M0M1A1M1A1A0
(b)	M1	Differential equation in *v* and *t* (in *x* and *t* for alternative solution)
	M1	Separate and integrate
	M1	Use limits on a definite integral or to find value of C'
	A1*	Obtain given result from correct working
		If mass is omitted from the differential equation can score M0M1M1A0

Question	Scheme	Marks	AOs
4(a)	$\frac{4}{3}\pi r^3 \times \frac{2}{3}r + \frac{2}{3}\pi r^3 \times \left(\frac{4}{3}r + \frac{3}{8}r\right) = \left(\frac{4}{3} + \frac{2}{3}\right)\pi r^3 \times d$	M1	2.1
	$\left(\frac{8}{9}r + \frac{8}{9}r + \frac{1}{4}r = 2d\right)$ $\left(\frac{73}{36}r = 2d\right)$	A1 A1	1.1b 1.1b
	$\Rightarrow d = \frac{73}{72}r$ *	A1*	2.2a
		(4)	
(b)			
	Resolving: $\leftrightarrow F = P$, $\updownarrow R = Mg$, $F_{max} = \mu R = \mu Mg$	M1	1.1b
	Slides if $P > \mu Mg$	A1	1.2
	Moments: $\frac{7}{3}rP = rMg$ Tilts if $P > \frac{3}{7}Mg$	B1	1.1b
	Comparison of restrictions to determine values of μ	M1	3.1a
	Slides first if $\mu Mg < \frac{3}{7}Mg$, $(0<)\mu < \frac{3}{7}$	A1	2.2a
		(5)	
			(9 marks)

Notes:		
(a)	M1	Moments equation. Dimensionally correct.
	A1 A1	Unsimplified equation with at most one slip Correct unsimplified equation
	A1*	Obtain given result from correct working.
(b)	M1	Resolve and use $F = \mu R$ to find values of P for sliding
	A1	Use the model to form the correct inequality
	B1	Correct inequality for tilting
	M1	Correct comparison of when it tilts and when it slides
	A1	Correct conclusion

Question	Scheme	Marks	AOs
5(a)			
	Resolve vertically	M1	3.4
	$\updownarrow\ 0.75g = T\cos\theta + R$ $\left(\frac{3g}{4} = \frac{2}{3}T + R\right)$	A1	1.1b
	Equation of motion	M1	3.4
	$\leftrightarrow\ 0.75 \times \sin\theta \times 9 = T\sin\theta$ $\left(0.75 \times \frac{\sqrt{20}}{10} \times 9 = T \times \frac{\sqrt{20}}{6}\right)$	A1	1.1b
	Complete strategy to find T and R	M1	3.1a
	$T = \frac{6 \times 0.75 \times 9}{10} = 4.05\ \text{(N)}$	A1	1.1b
	$R = 0.75g - \frac{2}{3}T = 4.65\ \text{(N)}$ or 4.7 (N)	A1	1.1b
		(7)	
(b)	Use $R = 0$ to form revised equations	M1	3.4
	$T\cos\theta = 0.75g$, $T\sin\theta = 0.75 \times \frac{10\sqrt{20}}{100}\omega^2$ $\left(\text{or } T\sin\theta = 0.75 \times 0.6\sin\theta \times \omega^2\right)$	A1	1.1b
	Complete strategy to find ω e.g. $\Rightarrow \tan\theta = \frac{\sqrt{20}\omega^2}{10g} = \frac{\sqrt{20}}{4}$	M1	1.1b
	$\omega = \sqrt{\frac{5g}{2}} = 4.95\ \text{(rad/s)}$	A1	1.1b
		(4)	
			(11 marks)

Notes:		
(a)	M1	Correct number of terms
	A1	Correct unsimplified equation
	M1	Circular motion. Condone confusion over units. $\frac{\sqrt{20}}{10}$ might not be seen as r cancels.
	A1	Correct unsimplified equation
	M1	Complete strategy to form sufficient equations to solve for T and R.
	A1	One force correct
	A1	Both correct (Finding value for R involves g)
(b)	M1	Correct interpretation of loss of contact
	A1	Revised equations
	M1	Solve for ω
	A1	Exact, 4.9 or 4.95 (non-exact answer requires substitution for g).

Question	Scheme	Marks	AOs
6(a)	Conservation of energy:	M1	3.1a
	$\frac{1}{2}mu^2 + mgl\sin\alpha = \frac{1}{2}mv^2 \quad \left(v^2 = \frac{9gl}{5} + 2gl\sin\alpha\right)$	A1	1.1b
	Equation of motion:	M1	3.1a
	$T - mg\sin\alpha = \frac{mv^2}{l}$	A1	1.1b
	Complete strategy to find T in terms of α	M1	2.1
	$\Rightarrow T = mg\sin\alpha + \frac{mv^2}{l} = mg\sin\alpha + \frac{9mg}{5} + 2mg\sin\alpha$ $= 3mg\sin\alpha + \frac{9mg}{5}$ *	A1*	2.2a
		(6)	
(b)	String slack $\Rightarrow T = 0 \Rightarrow \sin\alpha = -\frac{3}{5}$	B1	3.1a
	Use energy equation to find v:	M1	1.1b
	$v^2 = \frac{9gl}{5} - \frac{3}{5} \times 2gl, \quad v = \sqrt{\frac{3gl}{5}}$	A1	1.1b
		(3)	
(c)	Initial vertical component of speed $= \frac{4}{5} \times \sqrt{\frac{3gl}{5}}$	B1	1.1b
	Use of *suvat*: $0 = u^2 - 2gh = \frac{16}{25} \times \frac{3gl}{5} - 2gh$	M1	3.1a
	$h = \frac{24l}{125}$	A1	1.1b
	Total height above O $= \frac{3l}{5} + \frac{24l}{125} = \frac{99l}{125}$	A1	2.2a
		(4)	

Question	Scheme	Marks	AOs
(c) alt	Initial horizontal component of speed $=\frac{3}{5}\times\sqrt{\frac{3gl}{5}}$	B1	1.1b
	Conservation of energy:	M1	3.1a
	$mgh=\frac{1}{2}m\left(\frac{9}{5}\right)gl-\frac{1}{2}m\left(\frac{9}{25}\times\frac{3gl}{5}\right)$	A1	1.1b
	$h=\frac{99l}{125}$	A1	2.2a
		(4)	
			(13 marks)

Notes:

	M1	Must include all terms. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation
	M1	Must include all terms. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation
	M1	Complete strategy to form an expression for T in terms of α e.g. by using conservation of energy and the circular motion to form sufficient equations to obtain an equation in T only.
	A1*	Obtain given answer from correct working
(b)	B1	Correct deduction
	M1	Substitute value to find v^2
	A1	Correct only
(c)	B1	Correct vertical component of velocity when string goes slack.
	M1	Use of $v^2=u^2+2as$ or alternative complete method to find the additional height.
	A1	Additional height correct
	A1	Total height correct
(c) alt	B1	Correct horizontal component of velocity when string goes slack.
	M1	Use of conservation of energy or alternative complete method to find the height. All terms required. Condone sign errors.
	A1	Correct unsimplified equation in h and l
	A1	Correct answer

Question	Scheme	Marks	AOs
7(a)	Equation of motion about equilibrium position:	M1	3.1a
	$$\frac{4mg(x+e)}{l}-mg=-m\ddot{x}$$	A1	1.1b
	Extension e at equilibrium: $\frac{4mge}{l}=mg$, $\left(e=\frac{l}{4}\right)$	B1	1.1b
	$\Rightarrow \frac{4gx}{l}=-\ddot{x}$, $\left(\ddot{x}=-\frac{4g}{l}x\right)$	M1	3.1a
	This is of the form $\ddot{x}=-\omega^2 x$, so SHM *	A1*	3.2a
	Period $=\frac{2\pi}{\omega}$	M1	3.4
	$=2\pi\sqrt{\frac{l}{4g}}=\pi\sqrt{\frac{l}{g}}$ *	A1*	2.2a
		(7)	
7(a) alt	Equation of motion for extension x:	M1	3.1a
	$\frac{4mgx}{l}-mg=-m\ddot{x}$, $\ddot{x}=-\frac{4g}{l}\left(x-\frac{l}{4}\right)$	A1	1.1b
	Use substitution $X=x-\frac{l}{4}$	B1	1.1b
	$\Rightarrow \frac{4gX}{l}=-\ddot{X}$, $\left(\ddot{X}=-\frac{4g}{l}X\right)$	M1	3.1a
	This is of the form $\ddot{X}=-\omega^2 X$, so SHM *	A1*	3.2a
	Period $=\frac{2\pi}{\omega}$	M1	3.4
	$=2\pi\sqrt{\frac{l}{4g}}=\pi\sqrt{\frac{l}{g}}$ *	A1*	2.2a
		(7)	

Question	Scheme	Marks	AOs
(b)	Max speed $= a\omega \left(= \frac{l}{2}\sqrt{\frac{4g}{l}} \right)$	M1	3.4
	Max KE $= \frac{1}{2}m\left(\frac{l}{2}\sqrt{\frac{4g}{l}} \right)^2$	M1	1.2
	$= \frac{1}{2}m\frac{l^2}{4} \times \frac{4g}{l} = \frac{1}{2}mlg$	A1	1.1b
		(3)	
(c)	$x = a\cos\omega t = \frac{l}{2}\cos\sqrt{\frac{4g}{l}}t$	B1ft	2.2a
	Length of spring $< l \Rightarrow x = -\frac{l}{4}, \quad -\frac{l}{4} = \frac{l}{2}\cos\sqrt{\frac{4g}{l}}t$	M1	1.1b
	$\Rightarrow \sqrt{\frac{4g}{l}}t = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$, $t = \frac{\pi}{3}\sqrt{\frac{l}{g}}$ or $t = \frac{2\pi}{3}\sqrt{\frac{l}{g}}$	A1	1.1b
	Correct strategy	M1	3.1a
	Length of time $= \frac{2\pi}{3}\sqrt{\frac{l}{g}} - \frac{\pi}{3}\sqrt{\frac{l}{g}} = \frac{\pi}{3}\sqrt{\frac{l}{g}}$	A1	2.2a
		(5)	
			(15 marks)

Notes:		
(a)	M1	Equation of motion about equilibrium position. Need all terms. Dimensionally correct. Allow with their $e \neq 0$. Condone sign errors.
	A1ft	Correct unsimplified equation with *e* or their $e \neq 0$
	B1	Correct *e*
	M1	Complete strategy e.g. use equation of motion and equilibrium position to form equation in *x*.
	A1*	Reach given conclusion from correct working
	M1	Use the model to find periodic time (their ω)
	A1*	Obtain given answer from correct working
(b)	M1	Use the model to find the max speed. Follow their ω
	M1	Follow their a, ω
	A1	Correct simplified
(c)	B1	Or equivalent. Follow their a, ω
	M1	Follow their *e* and solve for *t*
	A1	One correct solution. Accept $t = \frac{2\pi}{3\omega}$, or $t = \frac{4\pi}{3\omega}$
	M1	Complete strategy to find the required interval: select formula for displacement as function of time and use symmetry of motion to find the time interval.
	A1	Correct answer from correct working