Mark Scheme Results

Summer 2023

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 01 Core Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$\{w = x+2 \Rightarrow\}\ x = w-2$	B1	3.1a
	$(w-2)^3 - 7(w-2)^2 - 12(w-2) + 6\ (=0)$	M1	1.1b
	$(w^3 - 6w^2 + 12w - 8) - 7(w^2 - 4w + 4) - 12(w-2) + 6$ $w^3 - 6w^2 + 12w - 8 - 7w^2 + 28w - 28 - 12w + 24 + 6$ $= w^3 + ...w^2 + ...w + ...$	M1	3.1a
	$w^3 - 13w^2 + 28w - 6 = 0$	A1 A1	1.1b 1.1b
		(5)	
	Alternative using sum, pair sum and product of roots:		
	$\alpha + \beta + \gamma = 7,\ \alpha\beta + \beta\gamma + \alpha\gamma = -12,\ \alpha\beta\gamma = -6$	B1	3.1a
	New sum: $\alpha + 2 + \beta + 2 + \gamma + 2 = (\alpha + \beta + \gamma) + 6 = 7 + 6 = 13$ New pair sum: $(\alpha+2)(\beta+2) + (\alpha+2)(\gamma+2) + (\beta+2)(\gamma+2)$ $= (\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 12 = -12 + 4\times7 + 12 = 28$ New product: $(\alpha+2)(\beta+2)(\gamma+2)$ $= \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 8$ $= -6 + 2\times-12 + 4\times7 + 8 = 6$	M1	3.1a
	$p = -"13",\ q = 28,\ r = -"6"$ **or** $w^3 - "13"w^2 + "28"w - "6"\ (=0)$	M1	1.1b
	$w^3 - 13w^2 + 28w - 6 = 0$	A1 A1	1.1b 1.1b
			(5 marks)

Notes:

Allow a variable other than *w* to be used for the first 4 marks.

<u>The "= 0" is not required until the final mark.</u>

B1: Selects the method of making a connection between x and w by writing $x = w-2$

M1: Applies the process of substituting their $x = "w-2"$ into the equation for all occurrences of x.

M1: Depends on having attempted substituting either $x = w-2$ or $x = w+2$ into the equation. This mark is for manipulating their resulting equation into the required form so must have gathered terms. Condone poor squaring/cubing of brackets as long as a cubic expression is obtained.

A1: At least two of p, q and r correct.

A1: Correct final <u>equation</u> (including "= 0"). **Must be an equation in *w*.**

Note if they say e.g. $x = w - 2$ and then substitute $w + 2$, it is possible to score B1 M0 M1

Note if they say e.g. $x = w + 2$ and then substitute $w - 2$, allow recovery

Alternative:

B1: Selects the method of giving three correct equations for the sum, pair sum and product in terms of α, β and γ. Note that the correct values may be seen embedded when they attempt the new sum, pair sum and product e.g. $(\alpha+2)(\beta+2)(\gamma+2) = \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 8$

$= \underline{-6} + 2(\underline{-12}) + 4(\underline{7}) + 8$

M1: Applies the process of finding the new sum, pair sum and product. Mark positively here and allow slips provided they are attempting $\alpha + 2 + \beta + 2 + \gamma + 2,\ (\alpha+2)(\beta+2) + (\alpha+2)(\gamma+2) + (\beta+2)(\gamma+2)$ and $(\alpha+2)(\beta+2)(\gamma+2)$

M1: In this method, this mark is for choosing p = – (their new sum), q = their new pair sum, r = – (their new product) **or** forming $w^2 - (\text{new sum})w^2 + (\text{new pair sum})w - (\text{new product})$

A1: At least two of p, q and r correct. As values or seen in their equation.

A1: Correct final <u>equation</u> (including "= 0"). **Must be an equation in *w*.**

In all methods, the final A mark depends on all the previous marks.

Question	Scheme	Marks	AOs
2(a)	$x^2+4x-5=(x+2)^2-9$	**B1**	1.1b
		(1)	
(b)	$\int \frac{1}{\sqrt{(x+p)^2-q}}\,dx = \text{arcosh}\left(\frac{x+p}{\sqrt{q}}\right)(+c)$ or $\ln\left(x+p+\sqrt{(x+p)^2-q}\right)(+c)$	**M1**	1.1a
	$=\text{arcosh}\left(\frac{x+2}{3}\right)$ or $\ln\left(x+2+\sqrt{(x+2)^2-9}\right)$ oe	**A1**	2.2a
		(2)	
(c)	Mean $=\frac{1}{13-3}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,dx$	**B1**	1.2
	$\frac{1}{10}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,dx=\frac{1}{10}\left(\text{arcosh}\left(\frac{15}{3}\right)-\text{arcosh}\left(\frac{5}{3}\right)\right)$ or $\frac{1}{10}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,dx=\frac{1}{10}\left(\ln\left(15+\sqrt{216}\right)-\ln\left(5+\sqrt{16}\right)\right)$	**M1**	1.1b
	$=\frac{1}{10}\ln\left(\frac{5+2\sqrt{6}}{3}\right)$ or $\frac{1}{20}\ln\left(\frac{49+20\sqrt{6}}{9}\right)$	**A1**	3.2a
		(3)	
		(6 marks)	

Notes:

(a)

B1: Correct completed square form. Allow 3^2 for 9.

(b)

M1: Achieves a correct form for the integration for their p and q from part (a):

$$\text{arcosh}\left(\frac{x+p}{\sqrt{q}}\right)(+c) \textbf{ or } \ln\left(x+p+\sqrt{(x+p)^2-q}\right)(+c) \textbf{ or e.g. } \ln\left(\frac{x+p}{\sqrt{q}}+\sqrt{\left(\frac{x+p}{\sqrt{q}}\right)^2-1}\right)(+c)$$

where $p\neq 0,\ q\neq 1$

Allow $\cosh^{-1}$ for arcosh

Allow attempts that use substitution following an attempt to complete the square but must be an appropriate substitution e.g. $x+p=\sqrt{q}\cosh u$ leading to a correct form as above.

A1: Correct integration. The "+ c" is not required. Apply isw once a correct expression is seen.

Note that $\ln\left(\frac{x+2}{3}+\sqrt{\left(\frac{x+2}{3}\right)^2-1}\right)(+c)$ is also correct

(c)

B1: Recalls the definition of a mean function accurately. $\frac{1}{13-3}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,\mathrm{d}x$ seen or implied.

Note that the $\frac{1}{13-3}$ may appear at the end. $\frac{1}{13-3}\int_3^{13}\mathrm{f}(x)\,\mathrm{d}x$ is sufficient as f (x) is defined in the question. Also allow it to be implied by e.g. $\frac{1}{10}\left[\mathrm{g}(x)\right]_3^{13}$ where g(x) is their integrated function.

M1: Applies the correct limits the right way round to whatever they think the answer to part (b) is.

This can be awarded if the $\frac{1}{10}$ is present or not.

A1: Correct answer in correct form. Allow equivalents e.g. $\frac{1}{10}\ln\left(\frac{5}{3}+\frac{2\sqrt{6}}{3}\right)$, $\frac{1}{20}\ln\left(\frac{49}{9}+\frac{20\sqrt{6}}{9}\right)$

And allow if the surd is not simplified e.g. $\frac{1}{10}\ln\left(\frac{5+\sqrt{24}}{3}\right)$, $\frac{1}{20}\ln\left(\frac{49+\sqrt{2400}}{9}\right)$

Apply isw once a correct answer is seen.

The brackets must be present in forms such as $\frac{1}{10}\ln\left(\frac{5}{3}+\frac{2\sqrt{6}}{3}\right)$, $\frac{1}{20}\ln\left(\frac{49}{9}+\frac{20\sqrt{6}}{9}\right)$ but not in e.g. $\frac{1}{10}\ln\frac{5+\sqrt{24}}{3}$

If extra values are offered then score A0

Question	Scheme	Marks	AOs
3(a)	e.g. $\lvert z_1\rvert=\sqrt{(-4)^2+4^2}$ or $\arg z_1=\pi-\frac{\pi}{4}$ oe	M1	1.1b
	$(z_1=)4\sqrt{2}\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)$ or e.g. $(z_1=)\sqrt{32}\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)$	A1	1.1b
		(2)	
(b)(i)	$\frac{z_1}{z_2}=\frac{"4\sqrt{2}"}{3}\left(\cos\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)+\mathrm{i}\sin\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)\right)=\ldots$ **or** $\frac{z_1}{z_2}=\frac{"4\sqrt{2}"\mathrm{e}^{"\frac{3\pi}{4}\mathrm{i}"}}{3\mathrm{e}^{\frac{17\pi}{12}\mathrm{i}}}=\frac{"4\sqrt{2}"}{3}\mathrm{e}^{\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)\mathrm{i}}$ **or** $\frac{z_1}{z_2}=\frac{-4+4\mathrm{i}}{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)}\times\frac{\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)+\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)}{\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)+\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)}=\ldots$	M1	3.1a
	$=-\frac{2\sqrt{2}}{3}-\frac{2\sqrt{6}}{3}\mathrm{i}$ or $-\frac{2\sqrt{2}}{3}-\mathrm{i}\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{2}}{3}+\mathrm{i}\left(-\frac{2\sqrt{6}}{3}\right)$	A1	1.1b
		(2)	

Notes

(a) <u>Correct answer with no working scores both marks in (a)</u>

M1: Any correct expression for $\lvert z_1\rvert$ or $\arg z_1$ e.g. $\lvert z_1\rvert=\sqrt{(-4)^2+4^2}$ or $\arg z_1=\pi-\frac{\pi}{4}$

A1: Correct expression. The $"z_1="$ is not required.

This mark is not for correct modulus and correct argument it is for the complex number written in the required form. Condone the missing closing bracket e.g. $(z_1=)\sqrt{32}\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right.$

(b)(i) <u>Correct answer with no working scores no marks in (b)(i)</u>

M1: Employs a correct method to find the quotient. E.g.

- uses modulus argument form and divides moduli and subtracts arguments the right way round
- uses exponential form and divides moduli and subtracts arguments the right way round
- converts z_2 to Cartesian form and multiplies numerator and denominator by the complex conjugate of the denominator. Allow if the "3" is missing for this method. Allow with decimals for this method e.g. $\frac{z_1}{z_2}=\frac{-4+4\mathrm{i}}{-0.258\ldots-0.965\ldots\mathrm{i}}\times\frac{-0.258\ldots+0.965\ldots\mathrm{i}}{-0.258\ldots+0.965\ldots\mathrm{i}}=\ldots$

<u>If they convert z_2 to Cartesian form it must be correct as shown or correct decimals.</u>

A1: Correct exact answer in the required form.

Do not allow e.g. $-\frac{2}{3}\left(\sqrt{2}+\sqrt{6}\mathrm{i}\right)$ or $\frac{-2\sqrt{2}-2\sqrt{6}\mathrm{i}}{3}$ unless a correct form is seen previously then apply isw.

Provided a correct method is shown as above, allow to go from the forms in the main scheme to the correct exact answer with no intermediate step.

(ii)	$z_2^4 = 3^4\left(\cos\left(4\times\frac{17\pi}{12}\right)+\mathrm{i}\sin\left(4\times\frac{17\pi}{12}\right)\right)$ **or** $(z_2)^4 = \left(3\mathrm{e}^{\frac{17\pi}{12}\mathrm{i}}\right)^4 = 3^4\mathrm{e}^{\frac{17\pi}{12}\times 4\mathrm{i}}$ **or** $z_2^4 = \left\{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)\right\}^4 = ...$	**M1**	1.1b
	$=\frac{81}{2}-\frac{81\sqrt{3}}{2}\mathrm{i}$ or $\frac{81}{2}-\mathrm{i}\frac{81\sqrt{3}}{2}$ or $\frac{81}{2}+\mathrm{i}\left(-\frac{81\sqrt{3}}{2}\right)$	**A1**	1.1b
		(2)	

(b)(ii) Correct answer with no working scores no marks in (b)(ii)

M1: Applies De Moivre's theorem correctly to z_2. E.g. uses polar form or exponential form and calculates the modulus as 3^4 and the argument as $4\times\frac{17\pi}{12}$

For attempts at $z_2^4 = \left\{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)\right\}^4$ you would need to see:

- the correct exact form used
- a clear and convincing attempt to expand the brackets e.g. by using a full binomial expansion or a complete attempt to multiply all 4 brackets together but you are not expected to check every detail
- a final answer in the required form with no obvious errors seen

So $z_2^4 = \left\{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)\right\}^4 = \frac{81}{2}-\frac{81\sqrt{3}}{2}\mathrm{i}$ scores no marks.

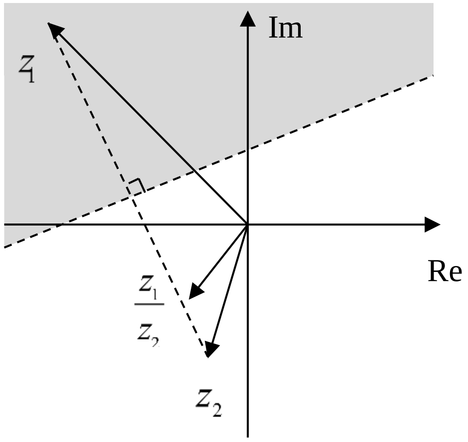
Similar guidance applies if they attempt to expand $\left\{3\left(\cos\frac{17\pi}{12}+\mathrm{i}\sin\frac{17\pi}{12}\right)\right\}^4$

A1: Correct exact answer in the required form.

Do not allow e.g. $\frac{81}{2}\left(1-\frac{81\sqrt{3}}{2}i\right)$ or $\frac{81-81\sqrt{3}\mathrm{i}}{2}$ unless a correct form is seen previously then apply isw.

Provided a correct method is shown as above, allow to go from the forms in the main scheme to the correct exact answer with no intermediate step.

(c)(i) and (ii)



Notes:		
(c)(i) **B1:** z_1 and z_2 correctly positioned. Look for <u>correct quadrants</u> with z_1 approximately on $y = -x$ and z_2 below $y = x$ closer to the origin than z_1. Note that the points are usually labelled but mark positively if it is clear which points are which if there is no labelling.	**B1**	1.1b
B1ft: $\frac{z_1}{z_2}$ in the correct quadrant. Follow through their answer to (b)(i). Note that the point is usually labelled but mark positively if it is clear which point it is. It is sometimes labelled as z_3 which is fine.	**B1ft**	1.1b
(ii) **M1:** Draws a line (solid or dashed) that is the perpendicular bisector of z_1z_2 **or** draws a line that crosses z_1z_2 and shades one of the sides of this line.	**M1**	3.1a
A1: A line drawn (solid or dashed) that is the perpendicular bisector of z_1z_2 **<u>with either side shaded</u>** as long as it is clear they are not discounting the upper region. The B1 in part (i) may not have been scored but z_1 must be in quadrant 2 and z_2 in quadrant 3. **Note that some candidates are drawing the region on a separate diagram and this is acceptable. You do not need to see a line joining z_1 to z_2.**	**A1**	1.1b
	(4)	
(10 marks)		

Question	Scheme	Marks	AOs
4	$\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}^n = \begin{pmatrix}1 & -2n\\0 & 1\end{pmatrix}$		
	$\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}^1 = \begin{pmatrix}1 & -2\times 1\\0 & 1\end{pmatrix}$ (so true when $n = 1$)	**B1**	2.2a
	(Assume true for $n = k$, then) $\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}^{k+1} = \begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix}$ or $\begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$	**M1**	2.1
	$= \begin{pmatrix}1 & -2k-2\\0 & 1\end{pmatrix}$ or $= \begin{pmatrix}1 & -2-2k\\0 & 1\end{pmatrix}$	**A1**	1.1b
	$= \begin{pmatrix}1 & -2(k+1)\\0 & 1\end{pmatrix}$	**A1**	2.2a
	Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all n.	**A1**	2.5
		(5)	
			(5 marks)

Notes:

B1: Shows true for $n = 1$.

Need to see $n = 1$ **substituted** into rhs. The minimum for B1 would be $\begin{pmatrix}1 & -2\times 1\\0 & 1\end{pmatrix} = \begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$.

There is no need to state "True for $n = 1$"

M1: (Assumes for $n = k$ and) multiplies original matrix by kth power matrix either way round. Note that the assumption statement is not needed for this mark (but see below) so just look for:

$$\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix} \text{ or } \begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$$

A1: Achieves a correct unsimplified matrix.

A1: Reaches correct form for the matrix with the $k + 1$ factored out with no errors and the correct unsimplified matrix seen previously. Note that the result may be proved by equivalence (see below).

A1: Correct conclusion **with assumption made** (which may be implied in their conclusion if they say "if true for $n = k$ then…"). This mark is dependent on all the previous marks apart from the B mark and is gained by conveying all the underlined points.

Allow this mark to score as long as all the underlined points are seen as narrative in their solution.

There must be the assumption statement somewhere <u>or</u> the "if…then…" idea in the conclusion. If awarded for the assumption statement condone e.g. true for $n = k$ in the conclusion.

The conclusion must convey the "if true for $n = k$ then true for $n = k + 1$" idea and not e.g. true for k, true for $k + 1$, true for 1 therefore…but see the previous note.

For the "true for all n" part condone e.g. "true for n", "true for all integers after 1", "true for $\mathbb{Z}^+$",
But do **not** allow "true for all values", "true for all real numbers"

<u>Q4 Extra Notes:</u>

1. For candidates who use *n* instead of *k* throughout withhold the final mark if the work is otherwise correct.

2. For equivalence proofs, this would be minimally acceptable:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2\times 1 \\ 0 & 1 \end{pmatrix} \quad \textbf{B1}$$

"Need to prove" oe e.g. "target is" , "$n = k + 1$", "need" etc. $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix}$

Note that there must be a reference to $k + 1$ as above or e.g. (Target =) $\begin{pmatrix} 1 & -2(k+1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix} \quad \textbf{M1A1A1}$$

Hence result is true for $n = k + 1$. As <u>true for $n = 1$</u> and have shown <u>if true for $n = k$ then it is true for $n = k + 1$</u>, so it is <u>true for all n</u>. **A1**

Without the "Need to prove…" oe the response would score B1M1A1 and then A1 if the factorised form was shown or equivalence shown and then A1 for the correct conclusion.

3. Allow e.g. "correct" for "true" in the conclusion

Question	Scheme	Marks	AOs
5(a)	$2(\lambda-5)+3(-3\lambda-4)-2(5\lambda+3)=6 \Rightarrow \lambda=...(-2)$ $\lambda="-2" \Rightarrow x=...$ or $y=...$ or $z=...$ or e.g. $2x+3(-3x-15-4)-2(5x+25+3)=6 \Rightarrow x=...$	M1	1.1b
	$(-7,\ 2, -7)$	A1	1.1b
		(2)	
(b)	E.g. $\mathbf{r}=\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2t\\3t\\-2t\end{pmatrix}$ meets the plane when $2(-5+2t)+3(-4+3t)-2(3-2t)=6 \Rightarrow t=...$	M1	3.1a
	$t=2 \Rightarrow$ mirror point is $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times4\\3\times4\\-2\times4\end{pmatrix}=...$	M1	1.1b
	$=\begin{pmatrix}3\\8\\-5\end{pmatrix}$	A1	1.1b
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+\mu\begin{pmatrix}3-(-7)\\8-2\\-5-(-7)\end{pmatrix}=...$	ddM1	1.1b
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+\mu\begin{pmatrix}10\\6\\2\end{pmatrix}$ *	A1*	2.1
		(5)	
	(b) Alternative for first 2 marks:		
	Distance from (–5, −4, 3) to plane is $\frac{\lvert 2\times-5+3\times-4-2\times3-6\rvert}{\sqrt{2^2+3^2+2^2}}=2\sqrt{17}$	M1	3.1a
	$\begin{vmatrix}2k\\3k\\-2k\end{vmatrix}=4\sqrt{17} \Rightarrow 4k^2+9k^2+4k^2=16\times17 \Rightarrow k=4$ $k=4 \Rightarrow$ mirror point is $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times4\\3\times4\\-2\times4\end{pmatrix}=...$	M1	1.1b

Do not penalise the omission of "r = " more than once in this question so penalise only once and on its first occurrence.

Notes:

(a) **Correct answer only scores no marks.**

M1: Substitutes the parametric form of the line into the plane and solves for their parameter and uses this to find at least one coordinate. Alternatively substitutes line equation into the plane equation to obtain and solve an equation in one variable.

A1: Correct point. Accept as $x = -7,\ y = 2,\ z = -7$ or as a vector.

(b)

M1: Identifies a point on l_1 and uses the equation of the line through this point perpendicular to the plane to find the parameter where it intersects the plane. May use a different starting point.

M1: Uses twice their parameter to find the image point of their starting point in the plane.

A1: Correct image point.

ddM1: Finds the equation of the line between their intersection point from (a) and their image point. May be implied if they use e.g. 3**i** + 8**j** – 5**k** as the position vector.
Alternatively, checks that the image point satisfies the equation for the other line.
Depends on both previous method marks.

A1*: Fully correct work with conclusion that hence the given equation gives the required line.
Must be as printed with "**r** = " but condone use of a different parameter e.g. t, λ etc.
If they checked that the image point satisfied the equation of line 2 they would need to then say e.g. that both lines pass through (–7, 2, –7) and make a minimal conclusion.

Alternative for first 2 marks in (b):

M1: Identifies a point on l_1 and finds the perpendicular distance from this point to the plane. May use a different starting point.

M1: Uses twice their distance to find the image point of their starting point in the plane.

(c)	Line joining mirror points intersects plane at $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times2\\3\times2\\-2\times2\end{pmatrix}$, so equation of line is $\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}-1-(-7)\\2-2\\-1-(-7)\end{pmatrix}=\ldots$	**M1**	3.1a
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}6\\0\\6\end{pmatrix}$ oe e.g. $\mathbf{r}=\begin{pmatrix}-1\\2\\-1\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$	**A1**	2.5
		(2)	
	Alternative 1 to (c) (Not on spec)		
	Normal to Π_2 : $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-3&5\\10&6&2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ Direction of l_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&4&3\\2&3&-2\end{vmatrix}=\begin{pmatrix}-17\\0\\-17\end{pmatrix}$ equation of line is $\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}-17\\0\\-17\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	
	Alternative 2 to (c) (Not on spec)		
	Normal to Π_2 : $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-3&5\\10&6&2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ $(-3\mathbf{i}+4\mathbf{j}+3\mathbf{k})\cdot(-7\mathbf{i}+2\mathbf{j}-7\mathbf{k})=8$ Π_2 is $3x-4y-3z=-8$ then e.g. solves simultaneously with Π_1 and $x=\lambda$ to give $y=2$, $z=\lambda$ So equation of line is $\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	
	Alternative 3 to (c) (Not on spec)		
	As alternative 2 to find the equation of plane 2: $3x-4y-3z=-8$ Then solves simultaneously with plane 1 to give e.g. $y=2$, $x=z$ Hence $\mathbf{r}=\begin{pmatrix}s\\2\\s\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	

Notes:

(c)

M1: A complete method to find the required equation. E.g. finds a second point on the common line and uses this and the point from (a) to find the direction and then forms the vector equation of the required line. May use earlier work to find the second point or start again.

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless this was already penalised in part (b).

Alternative 1:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 and then attempting the vector product of this with the normal to Π_1 to find the direction of the common line and forms the vector equation with a point on the line.
The normal vectors may also be found using scalar products e.g.

$$\mathbf{n} = x\mathbf{i} + y\mathbf{j} + \mathbf{k} \Rightarrow (x\mathbf{i} + y\mathbf{j} + \mathbf{k}).(\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) = 0, (x\mathbf{i} + y\mathbf{j} + \mathbf{k}).(10\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}) = 0 \Rightarrow x = ..., y = ...$$

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless already penalised in (b).

Alternative 2:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above and finds the equation of Π_2 by using a point on the line and then solves this with Π_1 by eliminating one of the variables to form the vector equation of the line of intersection.

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless already penalised in (b).

Alternative 3:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above and finds the equation of Π_2 by using a point on the line and then solves this simultaneously with Π_1 to form the vector equation of the line of intersection.

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless already penalised in (b).

There will be other methods in part (c)
Generally the method mark requires a <u>complete</u> attempt
to find the equation of the line of intersection.

(d)	Line from (c) must lie in plane, so $\begin{pmatrix}1\\0\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}1\\1\\a\end{pmatrix}$ $\Rightarrow \begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=0 \Rightarrow 1\times1+0\times1+1\times a=0 \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$b=\begin{pmatrix}-7\\2\\-7\end{pmatrix}.\begin{pmatrix}1\\1\\-1\end{pmatrix}=2$	**A1**	2.2a
		(3)	
	Alternative 1 to (d):		
	$\begin{pmatrix}-7\\2\\-7\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=b \Rightarrow -7+2-7a=b$ $\Rightarrow a=...$ or $b=...$ $\begin{pmatrix}-1\\2\\-1\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=b \Rightarrow -1+2-a=b$	**M1**	3.1a
	$a=-1$ or $b=2$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	
	Alternative 2 to (d) (Not on spec):		
	Normal to Π_2 : $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -3 & 5\\10 & 6 & 2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ $\begin{vmatrix}3 & -4 & -3\\2 & 3 & -2\\1 & 1 & a\end{vmatrix}=0 \Rightarrow 3(3a+2)+4(2a+2)-3(-1)=0 \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	
	Alternative 3 to (d):		
	$\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$ lies in $\Pi_3 \Rightarrow s+2+as=b$ $(a+1)s+2=b \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	

	Alternative 4 to (d):		
	Normal to Π_2 : $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 5 \\ 10 & 6 & 2 \end{vmatrix} = \begin{pmatrix} -36 \\ 48 \\ 36 \end{pmatrix}$ $(-3\mathbf{i}+4\mathbf{j}+3\mathbf{k})\cdot(-7\mathbf{i}+2\mathbf{j}-7\mathbf{k})=8$ Π_2 is $3x-4y-3z=-8$ then solves simultaneously with Π_1 and Π_3 and uses consistency e.g. $3x-4y-3z=-8$ $2x+3y-2z=6$ $x+y+az=b$ $x+y+az=b \Rightarrow x=b-y-az$ $3b-3y-3az-4y-3z=-8 \Rightarrow 7y+(3a+3)z=3b+8$ $2b-2y-2az+3y-2z=6 \Rightarrow y-(2a+2)z=6-2b$ Consistency $\Rightarrow -14a-14=3+3a \Rightarrow a=...$ or $3b+8=42-14b \Rightarrow b=...$	**M1**	3.1a
	$a=-1$ or $b=2$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a

(12 marks)

Notes:

(d)

M1: Realises the direction of (c) is perpendicular to the normal to Π_3 and applies the dot product = 0 to find *a*.

A1: Correct value for *a*

A1: Deduces the value of *b* using their *a* and one of their points on the line.

Alternative 1:

M1: Uses their point from (a) and another point on the line of intersection and substitutes both points into the given equation and solves the resulting equations for *a* or *b*.

A1: Correct value for *a* or *b*.

A1: Correct value for *a* and *b*.

Alternative 2:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above (may have been found earlier) and then attempts the determinant of the matrix of normal vectors = 0 and solves for *a*

A1: Correct value for *a*.

A1: Deduces the correct value of *b*. E.g. by using their *a* with a point on the line.

Alternative 3:

M1: Substitutes their line from part (c) into the equation for Π_3 and compares coefficients to establish a value for *a*.

A1: Correct value for *a*.
A1: Deduces the correct value of *b*.

Alternative 4:
M1: Finds the equation for Π_2 and then solves all 3 equations using the fact they are consistent leading to a value for *a* or a value for *b*.
A1: Correct value for *a* or *b*.
A1: Correct value for *a* and *b*.

There will be other methods in part (d)
Generally the method mark requires a <u>complete</u> attempt
to find the value of *a* or *b*.

Question	Scheme	Marks	AOs
6(a)	$\dfrac{dV}{dt}=3-\dfrac{4}{1+e^{0.8t}}\pm kV$ (where k is constant)	**M1**	3.3
	$t=0, V=10, \dfrac{dV}{dt}=-3 \Rightarrow -3=3-\dfrac{4}{1+1}-10k \Rightarrow k=...$	**dM1**	3.4
	$\Rightarrow 10k=4 \Rightarrow k=\dfrac{2}{5} \Rightarrow \dfrac{dV}{dt}=3-\dfrac{4}{1+e^{0.8t}}-0.4V$ *	**A1***	2.1
		(3)	
(b)	$\dfrac{d}{dt}\left(\arctan e^{0.4t}\right)=\dfrac{1}{1+\left(e^{0.4t}\right)^2}\times ke^{0.4t}$	**M1**	1.1b
	$\dfrac{d}{dt}\left(\arctan e^{0.4t}\right)=\dfrac{2e^{0.4t}}{5\left(1+e^{0.8t}\right)}$ oe	**A1**	1.1b
		(2)	
	Alternative to part (b):		
	$y=\arctan e^{0.4t} \Rightarrow \tan y=e^{0.4t} \Rightarrow \sec^2 y\dfrac{dy}{dx}=0.4e^{0.4t}$	**M1**	1.1b
	$\dfrac{dy}{dx}=\dfrac{0.4e^{0.4t}}{\sec^2 y}=\dfrac{0.4e^{0.4t}}{1+\tan^2 y}=\dfrac{0.4e^{0.4t}}{1+\left(e^{0.4t}\right)^2}$	**A1**	1.1b
		(2)	
(c)	$\dfrac{dV}{dt}+0.4V=3-\dfrac{4}{1+e^{0.8t}} \Rightarrow I.F.\left(=e^{\int 0.4\,dt}\right)=e^{0.4t}$	**B1**	2.2a
	$e^{0.4t}V=\displaystyle\int 3e^{0.4t}-\dfrac{4e^{0.4t}}{1+e^{0.8t}}\,dt$	**M1**	1.1b
	$=Ae^{0.4t}-B\arctan\left(e^{0.4t}\right)(+c)$	**M1**	1.1b
	$e^{0.4t}V=7.5e^{0.4t}-10\arctan\left(e^{0.4t}\right)\ (+c)$	**A1**	1.1b
	$V=10, t=0 \Rightarrow 10=7.5-10\arctan 1+c \Rightarrow c=...$	**M1**	3.4
	$V=7.5-10e^{-0.4t}\arctan\left(e^{0.4t}\right)+2.5(\pi+1)e^{-0.4t}$	**A1**	2.1
		(6)	
(d)	E.g. $V(10)\approx 7.4$ litres so the model is not very accurate as it predicts approximately 7.5% below the actual level.	**B1ft**	3.5a
		(1)	
		(12 marks)	

Notes

(a)

M1: Sets up the correct equation for the model using the information in the question.

dM1: Uses the initial conditions to find the constant of proportionality for flow out.

Condone use of $\frac{dV}{dt} = +3$. **Depends on the first mark.**

A1*: Correct equation shown from correct work proceeding via $10k = 4$ to find k.

Attempts in (a) using verification score no marks:

E.g. $\frac{dV}{dt} = 3 - \frac{4}{1+e^{0.8t}} - \frac{2}{5}V \Rightarrow -3 = 3 - \frac{4}{2} - 0.4V \Rightarrow V = \frac{4}{0.4} = 10$

(b)

M1: Differentiates to achieve the form shown. Allow $k = 1$

A1: Correct derivative in any form. Need not be simplified.

Alternative:

M1: Takes tan of both sides and differentiates implicitly and reaches $\frac{1}{1+\left(e^{0.4t}\right)^2} \times ke^{0.4t}$. Allow $k = 1$.

A1: Correct derivative in any form. Need not be simplified.

(c)

B1: Deduces the correct integrating factor for the equation. May be implied by sight of $\frac{d}{dt}\left(e^{0.4t}V\right) = ...$ or equivalent work.

M1: Fully multiplies through by their integrating factor and integrates the LHS (look for $I.F. \times V = \int I.F. \times \left(3 - \frac{4}{1+e^{0.8t}}\right) dt$ though condone missing dt.

M1: Attempts the integral of the RHS.

Award for $\int \alpha e^{0.4t}\, dt = \beta e^{0.4t}$ $\alpha \neq \beta$ or $\int \frac{\alpha e^{0.4t}}{1+e^{0.8t}} dt = \beta \arctan e^{0.4t}$, $\beta \neq 0$

A1: Correct integration, need not be simplified. Allow if the + c is missing for this mark.

M1: Attempts to find their constant – which must have been treated correctly from point of integration. Note that this is not formally dependent but there must have been an attempt to integrate.

A1: Correct answer. The question says "simplest form" but allow equivalent expressions e.g.

$V = 7.5 - \frac{10\arctan\left(e^{0.4t}\right)}{e^{0.4t}} + \frac{5\pi}{2e^{0.4t}} + \frac{5}{2e^{0.4t}}$ but do not allow inexact values for the constants.

(d)

B1ft: Evaluates V where $V > 0$ at $t = 10$ and makes an appropriate comment.

For the evaluation, allow if a value of V is obtained even if there is no evidence of substitution provided that it is clear that $t = 10$ has not been substituted into something that is not V. So you do not need to check their value.

For the tolerance you may need to use your own judgement but a general guide is:

$0 < V < 7$	Not a good model
$7 \leqslant V < 7.7$ or $8.3 \leqslant V < 9$	Allow good **or** poor model
$7.7 \leqslant V < 8.3$	Good model
$V \geqslant 9$	Not a good model

Question	Scheme	Marks	AOs
7(a)	All the even terms are positive and all the odd ones are negative. or $\sum_{r=1}^{2n}(-1)^r \mathrm{f}(r) = -\mathrm{f}(1)+\mathrm{f}(2)-\mathrm{f}(3)+\mathrm{f}(4)-\ldots$	**M1**	2.4
	$\sum_{r=1}^{2n}(-1)^r \mathrm{f}(r) = -\mathrm{f}(1)+\mathrm{f}(2)-\mathrm{f}(3)+\mathrm{f}(4)-\ldots-\mathrm{f}(2n-1)+\mathrm{f}(2n)$ $= \mathrm{f}(2)+\mathrm{f}(4)+\ldots+\mathrm{f}(2n)-\left(\mathrm{f}(1)+\mathrm{f}(3)+\ldots+\mathrm{f}(2n-1)\right)$ $= \sum_{r=1}^{n}\left(\mathrm{f}(2r)-\mathrm{f}(2r-1)\right)$ *	**A1***	3.1a
		(2)	
(b)	$\sum_{r=1}^{2n} r\left((-1)^r+2r\right)^2 = \sum_{r=1}^{2n} r\left(1+4r(-1)^r+4r^2\right)$	**M1**	2.1
	$= \underline{\frac{1}{2}(2n)(2n+1)+4\frac{(2n)^2}{4}(2n+1)^2} + 4\sum_{r=1}^{2n}(-1)^r r^2$	**M1**	1.1b
	$\sum_{r=1}^{2n}(-1)^r r^2 = \sum_{r=1}^{n}\left((2r)^2-(2r-1)^2\right)$	**M1**	3.1a
	$\sum_{r=1}^{2n} r\left((-1)^r+2r\right)^2 = n(2n+1)+4n^2(2n+1)^2+4\underline{\left(4\frac{n}{2}(n+1)-n\right)}$	**B1**	1.1b
	$= n(2n+1)+4\left(n(2n+1)\right)^2+4n(2n+1)$ $= n(2n+1)\left(1+4n(2n+1)+4\right)$	**dM1**	2.1
	$= n(2n+1)\left(8n^2+4n+5\right)$ *	**A1***	1.1b
		(6)	
(c)	$\sum_{r=14}^{50} r\left((-1)^r+2r\right)^2 = \sum_{r=1}^{50} r\left((-1)^r+2r\right)^2 - \sum_{r=1}^{13} r\left((-1)^r+2r\right)^2$	**M1**	1.1b
	$= \sum_{r=1}^{50} r\left((-1)^r+2r\right)^2 - \sum_{r=1}^{12} r\left((-1)^r+2r\right)^2 - 13\times 25^2$ or $= \sum_{r=1}^{50} r\left((-1)^r+2r\right)^2 - \sum_{r=1}^{14} r\left((-1)^r+2r\right)^2 + 14\times 29^2$	**M1**	3.1a
	$= 25\times 51\times 5105 - 6\times 13\times 317 - 13\times 25^2$ $(= 6508875 - 24726 - 8125)$ or $25\times 51\times 5105 - 7\times 15\times 425 + 14\times 29^2$ $(= 6508875 - 44625 + 11774)$	**M1**	2.1
	$= 6476024$	**A1**	1.1b
		(4)	
		(12 marks)	

Notes:

(a)

M1: This mark is for stating that all the odd terms are negative (or subtracted) and all the even terms are positive (or added) or for showing this by writing down at least 4 terms as above. May be achieved via rhs e.g. $\sum_{r=1}^{n}\left(\mathrm{f}(2r)-\mathrm{f}(2r-1)\right)=\mathrm{f}(2)-\mathrm{f}(1)+\mathrm{f}(4)-\mathrm{f}(3)+\ldots$

A1*: A full and convincing argument that shows the equivalence of both sides of the equation. E.g. clearly demonstrates how the terms separate as $\mathrm{f}(2)+\mathrm{f}(4)+\ldots+\mathrm{f}(2n)$ for the even powers and $\mathrm{f}(1)+\mathrm{f}(3)+\ldots+\mathrm{f}(2n-1)$ for the odd powers and concludes $\sum_{r=1}^{n}\left(\mathrm{f}(2r)-\mathrm{f}(2r-1)\right)$*

Need to see:

- the even terms grouped as e.g. $\mathrm{f}(2)+\mathrm{f}(4)+\ldots+\mathrm{f}(2n)$
- the odd terms grouped as e.g. $\mathrm{f}(1)+\mathrm{f}(3)+\ldots+\mathrm{f}(2n-1)$
- a conclusion

See below for some examples.

(b)

M1: Expands summand fully and applies $(-1)^{2r}=1$ or $r(-1)^{2r}=r$ at some point in their solution. Need not split the summation for this mark, $\sum r\left(1+k(-1)^r r+mr^2\right)$ can be accepted where k and m are 2 or 4.)

M1: Applies the correct formulae for sum of integers and sum of cubes to the r and mr^3 terms with $2n$

M1: Applies result of part (a) to the $(-1)^r$ term. Condone minor slips but the structure should be correct e.g.

$\lambda\sum_{r=1}^{2n}(-1)^r r^2=\sum_{r=1}^{n}\left(\alpha(2r)^2-\beta(2r-1)^2\right)$ but **not** e.g. $\lambda\sum_{r=1}^{2n}(-1)^r r^2=\sum_{r=1}^{n}\left(\alpha(2r)^2-\beta\left((2r)^2-1\right)\right),\ \alpha,\beta>0$

B1: Correct expression for $4\sum_{r=1}^{2n}(-1)^r r^2$ which may be unsimplified e.g. $4\left(4\frac{n}{2}(n+1)-n\right)$ as shown or e.g. $8n^2+4n,\ \frac{16}{2}n(n+1)-4n,$ etc. **Depends on previous method mark.**

dM1: Depends on all previous M marks. Takes out a factor of $n(2n+1)$ from a quartic expression. To score this mark, there must be a factor of n and an obvious factor of $(2n+1)$ if they leave the expressions inside the outer brackets factorised. However, this mark can be scored if they expand fully, take out a factor of n and then attempt to e.g. divide by $(2n+1)$ or e.g. use inspection to determine the quadratic factor. Must reach at least $=n(2n+1)\left(An^2+\ldots n\right)$ where A is the coefficient of their cubic expression.

A1*: Correct answer obtained with no errors seen and suitable intermediate steps shown. Note that it is acceptable to go from $n\left(16n^3+16n^2+14n+5\right)$ to $n\left(2n+1\right)\left(8n^2+4n+5\right)$

(c)

M1: Attempts to split the sum to a difference upper limit 50 and lower limit 13. May be implied. Condone e.g. $\sum_{r=14}^{50}r\left((-1)^r+2r\right)^2=\sum_{r=1}^{50}\mathrm{f}(r)-\sum_{r=1}^{13}\mathrm{f}(r)$ or even $\sum_{r=14}^{50}r\left((-1)^r+2r\right)^2=\sum_{r=1}^{50}\ -\sum_{r=1}^{13}$

M1: Splits expression with **even** upper limits for both sums and evaluates an appropriate balancing term. The balancing term must be correct for their method but ignore whether it is added or subtracted for this mark.

M1: Attempts to use the result from (b) at least once **correctly** with an integer – so using n = 25 with upper limit 50, or n = 6 with upper limit 12 e.g. $25\times51\times5105$ or $6\times13\times317$. **Use of their upper limits in the formula is M0**

A1: 6476024 from correct work. Question says "hence" so use of the result from (b) must be clear.

Question	Scheme	Marks	AOs
8(a)	Accept E.g. • 1 month is too old for "newborn" • The mammals might not start breeding at exactly 3 months old • The mammals will stop breeding beyond a certain age • Being over 3 months old doesn't necessarily mean the mammal can breed • Some mammals over 3 months may be infertile so will not be breeders • Some juveniles might be breeders But not • The size of the categories is different • There might be overlap • The exact age of mammals might not be known • The numbers in each category will be different • Breeding age is different for different species	**B1**	3.5b
		(1)	

(a)
B1: Any valid limitation – see scheme for some examples. Must refer to a feature of the categories given.

(b)(i)	$\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}^2\begin{pmatrix}0\\0\\k\end{pmatrix}=\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}2k\\0\\0.96k\end{pmatrix}$ or $\begin{pmatrix}0&0.96&1.92\\ab&b^2&2a\\0.48a&0.48b+0.96\times0.48&0.96^2\end{pmatrix}\begin{pmatrix}0\\0\\k\end{pmatrix}=\begin{pmatrix}1.92k\\2ak\\0.9216k\end{pmatrix}$	**M1**	3.4
	$48=2\times0.96k\Rightarrow k=...$	**dM1**	1.1b
	$k=25$ so 25 mammals at the start of the study	**A1**	3.2a
(ii)	$40=2ka\Rightarrow a=0.8$*	**A1***	1.1b
		(4)	

(b)(i)

M1: Attempts to use the given information to set up a matrix equation and find the numbers of mammals after one month e.g.

$$\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}N_0\\J_0\\B_0\end{pmatrix}=\begin{pmatrix}2B_0\\aN_0+bJ_0\\0.48J_0+0.96B_0\end{pmatrix}$$

or attempts to square the matrix to find the number of mammals after two months e.g.

$$\begin{pmatrix}0&0.96&1.92\\ab&b^2&2a\\0.48a&0.48b+0.96\times0.48&0.96^2\end{pmatrix}\begin{pmatrix}N_0\\J_0\\B_0\end{pmatrix}=...$$

dM1: Forms an equation, in their variable for number of breeders at the start, setting their number of newborns after 2 months equal to 48 and solves for their variable to find the initial number of breeders.

A1: For identifying 25 mammals at the start of the study. Allow 25 mammals or just 25 or e.g. $B_0=25$ so ignore how they label it just look for 25

Note that in some cases work may be minimal e.g.

$$\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}0\\0\\B_0\end{pmatrix}=\begin{pmatrix}N_1\\J_1\\B_1\end{pmatrix}\Rightarrow 0.96B_0=B_1,\quad \begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}N_1\\J_1\\B_1\end{pmatrix}=\begin{pmatrix}48\\40\\B_2\end{pmatrix}\Rightarrow 2B_1=48$$

$$B_1=24=0.96B_0\Rightarrow B_0=25$$

(ii)

A1*: For correctly showing a = 0.8. Must see the correct work to establish the correct value or equivalent by verification with a minimal conclusion e.g.

$$\begin{pmatrix}0&0&2\\0.8&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}0\\0\\25\end{pmatrix}=\begin{pmatrix}50\\0\\24\end{pmatrix}=\begin{pmatrix}48\\40\\B_2\end{pmatrix}\Rightarrow 0.8\times50=40 \text{ Hence true *}$$

(c)	$\det\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} = 0 - 0 + 2(0.48\times0.8-0) = 0.768$	**B1**	2.2a
	$\text{adj}\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} = \begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$	**M1**	1.1b
	$= \begin{pmatrix} 1.25b & 1.25 & -\frac{125}{48}b \\ -1 & 0 & \frac{25}{12} \\ 0.5 & 0 & 0 \end{pmatrix}$ oe e.g. $\frac{1}{0.768}\begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$ or e.g. $\frac{125}{96}\begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$	**A1**	1.1b
		(3)	

(c)

B1: Deduces correct determinant for the matrix. Allow equivalents e.g. $\frac{96}{125}$ May be implied.

M1: Recognisable attempt at the adjoint matrix. Look for at least 3 non-zero entries correct.

A1: Correct inverse. Accept awrt $-2.6b$ for the upper right entry and awrt 2.08 for middle right entry, or accept with determinant still outside. Apply isw once a correct answer is seen.

(d)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.25b & 1.25 & -\frac{125}{48}b \\ -1 & 0 & \frac{25}{12} \\ 0.5 & 0 & 0 \end{pmatrix} \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ $\text{Total} = 1.25b \times 596 + 1.25 \times 464 - \frac{125}{48}b \times 437 - 596 + \frac{25}{12} \times 437 + 0.5 \times 596$	**M1**	3.1b
	$\Rightarrow 1015 = x + y + z = 745b + 580 - 1138b - 596 + 910.4 + 298 \Rightarrow b = \ldots$	**dM1**	3.4
	$b = \text{awrt } 0.45$	**A1**	1.1b
		(3)	
	(d) Alternative:		
	$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix} \Rightarrow \begin{matrix} 2z = 596 \\ 0.8x + by = 464 \\ 0.48y + 0.96z = 437 \end{matrix}$ $\Rightarrow z = 298, y = \frac{3773}{12} (314.4\ldots)$ $x + y + z = 1015 \Rightarrow x = \ldots \frac{4831}{12} (402.5\ldots)$	**M1**	3.1b
	$0.8x + by = 464 \Rightarrow 0.8 \times \frac{4831}{12} + b \times \frac{3773}{12} = 464 \Rightarrow b = \ldots$	**dM1**	3.4
	$b = \text{awrt } 0.45$	**A1**	1.1b
		(3)	

(d)

M1: Attempts (their inverse matrix) $\times \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ correctly and adds the 3 expressions together to find the total in terms of *b*.

M1: Sets their total = 1015 and solves for *b*.

A1: awrt 0.45

Alternative:

M1: Uses the original matrix with $a = 0.8$ and $\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ to form 3 equations in their variables and *b* and uses these and the 1015 to find the number of Newborns.

M1: Uses their values in the *y* component and solves for *b*.

A1: awrt 0.45

(e)	Let NM_n be newborn males and NF_n be newborn females in month n $$\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0.84 \\ 0 & 0 & 0 & 1.16 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$$ or e.g. $$\begin{pmatrix} NF_{n+1} \\ NM_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1.16 \\ 0 & 0 & 0 & 0.84 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NF_n \\ NM_n \\ J_n \\ B_n \end{pmatrix}$$	**M1** **A1ft**	3.5c 3.3
		(2)	
			(13 marks)

Notes:

(e)

M1: Defines new variables for male and female newborns (accept if a clear notation is used if not defined) and sets up a 4×4 matrix with structure shown, or male and female rows swapped, with the correct 0 entries in at least 4 places.

A1ft: Fully correct matrix system shown, accepting anything (including 0) for the unknown spaces shown – but must have all the 0’s **and** upper right entries correct. Accept *b* or their value of *b* in place of 0.45

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