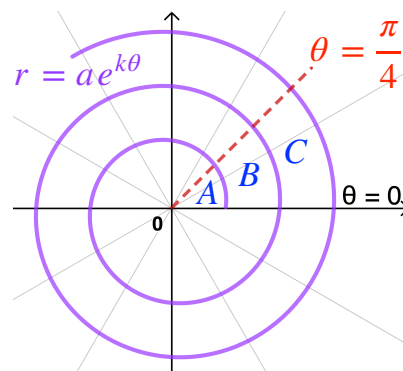


Area enclosed by two curves

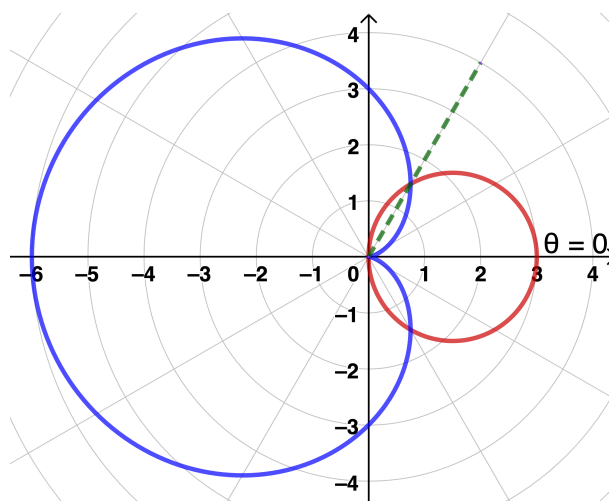
Starter

1. **(Review of last lesson)**

The diagram shows the function $r = ae^{k\theta}$, where a and k are positive constants. Prove that the areas A , B and C formed by the lines $\theta = 0$, $\theta = \frac{\pi}{4}$ and the spiral form a geometric sequence and find the common ratio.



2. Using the graph as a guide, find the area enclosed between the curves $r = 3 - 3 \cos \theta$ and $r = 3 \cos \theta$. Give your answer exactly.



Notes

As you would expect, we need to **find the points of intersection** before we can find the area between curves expressed in polar form. A **sketch of the curves** indicates which areas need to be found.

With polar coordinates we are finding the **area between the curve and two half-lines** from the origin, not two vertical lines as with Cartesian coordinates.

E.g. 1 Sketch the curves $r = 3 \sin \theta$ and $r = 4 \sin 2\theta$ on the same diagram. Hence find the area enclosed between these curves. Give your answer to 3 s.f..

Video: [Area bounded by a cardioid and a loop](#)

[Area bounded by a polar curve EQ](#)

[Solutions to Starter and E.g.s](#)

Exercise

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Summary

With polar coordinates we are finding the area **between the curves and half-lines** from the origin, not two vertical lines as with Cartesian coordinates. A sketch of the curve is very important to know what to calculate.