

Averages and measures of spread of continuous random variables

Starter

1. **(Review of last lesson)**

The life, X , of the StayBrite light bulb is modelled by the probability density function

$$f(x) = \begin{cases} ke^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}, \text{ where } X \text{ is measured in thousands of hours.}$$

- Find k .
- Find the probability that a StayBrite bulb lasts longer than 1000 hours.
- Find the probability that a StayBrite bulb lasts less than 500 hours.

Notes

The expectation or mean of a continuous random variable, X , is defined by:

$$E(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$$

The variance of a continuous random variable, X , is defined by:

$$\text{Var}(X) = \sigma^2 = \int_{-\infty}^{\infty} x^2f(x)dx - \mu^2$$

These formulae appear in the formula booklet.

From AS level, discrete random variables used $\text{Var}(X) = E(X^2) - E^2(X)$ and this is also true for

crv with $E(X^2) = \int_{-\infty}^{\infty} x^2f(x)dx$

E.g. 1 The probability density function $f(x) = \begin{cases} 2x - 4 & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$.

Find the mean and variance.

Working: Mean = $\int_{-\infty}^{\infty} xf(x)dx = \int_2^3 x(2x - 4)dx = \frac{8}{3}$

$$\begin{aligned} \text{Variance} &= \int_{-\infty}^{\infty} x^2f(x)dx - \mu^2 \\ &= \int_2^3 x^2(2x - 4)dx - \left(\frac{8}{3}\right)^2 \\ &= \frac{1}{18} \end{aligned}$$

E.g. 2 The EverOn torch battery has a life of X hours. The variable X is modelled by the

probability density function $f(x) = \begin{cases} 3000x^{-4} & x \geq 10 \\ 0 & \text{otherwise} \end{cases}$

Find the mean and standard deviation of EverOn torch batteries.

Mode and median of a continuous random variable

The **mode** of a continuous random variable is the **x -value** that gives the probability density function its **maximum value** over the given domain. Make sure you check the **end points** as these could give a higher point than a maximum turning point. A sketch of the graph can help.

The **median**, m , of a continuous random variable is the value such that $P(X \leq m) = \frac{1}{2}$.

i.e.
$$\int_{-\infty}^m f(x)dx = \frac{1}{2}$$

E.g. 3 The crv X has pdf where $f(x) = \begin{cases} \frac{3}{80}(2+x)(4-x) & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$.

Find the mode and median.

Working: The graph indicates that the mode will be the maximum value of the curve.

Given that the roots occur when $x = -2$ and $x = 4$, the maximum will occur when $x = 1$.

So the mode is 1.

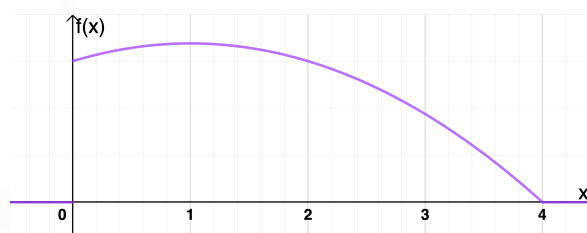
Median,
$$\int_{-\infty}^m f(x)dx = \frac{1}{2}:$$

$$\int_0^m (8 + 2x - x^2)dx = \frac{40}{3}$$

$$24m + 3m^2 - m^3 = 40 \Rightarrow$$

Since $0 \leq m \leq 4$, $m \approx 1.52$

The mode is 1 and the median is 1.52 (3 s.f.).



E.g. 4 A continuous random variable is modelled by $f(x) = \begin{cases} \frac{1}{6}(x^2 - 4x + 5) & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$.

Find the mode and median.

Video: [Mean and variance of continuous random variable](#)

[Solutions to Starter and E.g.s](#)

Exercise

p125 7B Qu 1i, 2i, 3-7 (8 red)

Summary

Mean: $E(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\text{Var}(X) = \sigma^2 = \int_{-\infty}^{\infty} x^2f(x)dx - \mu^2$...or... $\text{Var}(X) = E(X^2) - E^2(X)$

Mode: the x -value that gives the pdf its **maximum value** over the given domain.
Check the **end points**.

Median, m : $P(X \leq m) = \frac{1}{2}$ i.e. $\int_{-\infty}^m f(x)dx = \frac{1}{2}$