Centre of mass of a system of point masses

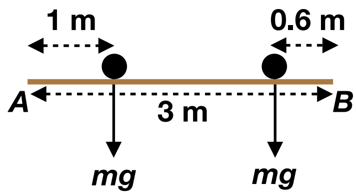
Starter

1. (Review of A2 Ma work) By considering moments about A, find the distance of the centre of mass of these systems from A.
   (a) A light rod AB of length 3 m has identical masses placed 1 m from A and 0.6 m from B.
   (b) A light rod AB of length 5 m has a mass of 4 kg placed 1.5 m from A and another mass of 3 kg placed 1.8 m from B.
   (c) A light rod AB of length 1.8 m has a mass of 8 kg placed at A, a mass of 5 kg placed 1.2 m from A and a mass of 4 kg placed at B.

Working: (a) Let the masses of A and B be m.
Let $\bar{x}$ be the distance of the CoM from A

$$2mg\bar{x} = mg \times 1 + mg \times 2.4$$
$$2\bar{x} = 3.4$$
$$\bar{x} = 1.7$$

The centre of mass is 1.7 m from A.



Notes

From the starter, it can be seen that g cancels each time. Therefore, when finding the centre of mass of an object, we just need to consider the ***individual masses*** of the system.

1-dimensional (i.e. rods)

Problems can be solved by considering the equivalent moment using the sum of the masses.

Let $\bar{x}$ be the distance of the centre of mass from a ***chosen fixed point*** (usually one of the ends) and let x_1, x_2, etc be the distances of the masses m_1, m_2, etc from the chosen fixed point.

$$(m_1 + m_2 + m_3 + \ldots)\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \ldots$$

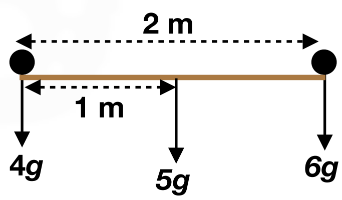
Let M be the sum of the masses then $$M\bar{x} = \sum_{i=1}^{n} m_ix_i$$

E.g. 1 A uniform rod of length 2 m has mass 5 kg. Masses of 4 kg and 6 kg are fixed at each end of the rod. Find the centre of mass of the system.

Working: Let $\bar{x}$ be the distance of the CoM from the 4 kg mass
$\circlearrowright$ about 4 kg mass:

$$(4 + 5 + 6)\bar{x} = 6 \times 2 + 5 \times 1$$
$$15\bar{x} = 17$$
$$\bar{x} = \frac{17}{15} = 1.1\dot{3}$$

The centre of mass is $1.1\dot{3}$ m from the 4 kg mass



E.g. 2 A rod, AB, of mass 1.1 kg and length 1.2 m has its centre of mass 0.48 m from the end A. What mass should be attached to the end B to ensure that the centre of mass is at the midpoint of the rod?

2-dimensional (i.e. uniform laminas)

In 2-dimensions, the centre of mass $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$ relative to a fixed point (usually the origin) of a series of masses m_1, m_2, etc whose centre of masses are at $\begin{pmatrix}x_1\\ y_1\end{pmatrix}$, $\begin{pmatrix}x_2\\ y_2\end{pmatrix}$ etc. is given by:

$$M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m_1\begin{pmatrix}x_1\\ y_1\end{pmatrix} + m_2\begin{pmatrix}x_2\\ y_2\end{pmatrix} + \ldots m_n\begin{pmatrix}x_n\\ y_n\end{pmatrix}$$

E.g. 3 Masses of m, $2m$ and $3m$ are placed at the points $(4,\ 3)$, $(1,\ 1)$ and $(3,\ 0)$. Find the centre of mass of this system.

Working: $\circlearrowright$ about the origin:

$$6m\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m\begin{pmatrix}4\\ 3\end{pmatrix} + 2m\begin{pmatrix}1\\ 1\end{pmatrix} + 3m\begin{pmatrix}3\\ 0\end{pmatrix}$$
$$6\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}15\\ 5\end{pmatrix}$$
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}2.5\\ \frac{5}{6}\end{pmatrix}$$

E.g. 4 Find the centre of mass of a body comprising of a uniform square plate of mass 3 kg and side length 2 m with small objects of mass 1 kg, 2 kg, 4 kg and 5 kg at the corners of the square. Assume the 1 kg is at the origin with the other masses placed anti-clockwise around the square.

Video: **CoM (particles in a line)**
Video: **CoM (particles in a plane)**
Video (password needed): **Introduction to centres of mass**

Solutions to Starter and E.g.s

Exercise
p107 5A Qu 1-9

Summary
1-dimensional (i.e. rods)
$(m_1 + m_2 + m_3 + \ldots)\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \ldots$ where m_i are the masses and x_i are the distances from of the masses to the chosen fixed point.

Let M be the sum of the masses then $$M\bar{x} = \sum_{i=1}^{n} m_ix_i$$

2-dimensional (i.e. uniform laminas)
$$M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m_1\begin{pmatrix}x_1\\ y_1\end{pmatrix} + m_2\begin{pmatrix}x_2\\ y_2\end{pmatrix} + \ldots m_n\begin{pmatrix}x_n\\ y_n\end{pmatrix}$$