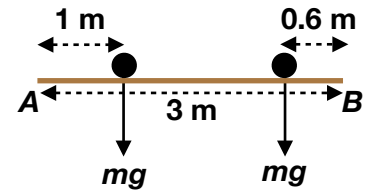


Centre of mass of a system of point masses

Starter

1. **(Review of A2 Ma work)** By considering moments about A , find the distance of the centre of mass of these systems from A .
- A light rod AB of length 3 m has identical masses placed 1 m from A and 0.6 m from B .
 - A light rod AB of length 5 m has a mass of 4 kg placed 1.5 m from A and another mass of 3 kg placed 1.8 m from B .
 - A light rod AB of length 1.8 m has a mass of 8 kg placed at A , a mass of 5 kg placed 1.2 m from A and a mass of 4 kg placed at B .

Working: (a) Let the masses of A and B be m .
 Let \bar{x} be the distance of the CoM from A
 $2mg\bar{x} = mg \times 1 + mg \times 2.4$
 $2\bar{x} = 3.4$
 $\bar{x} = 1.7$
 The centre of mass is 1.7 m from A .



Notes

From the starter, it can be seen that g cancels each time. Therefore, when finding the centre of mass of an object, we just need to consider the **individual masses** of the system.

1-dimensional (i.e. rods)

Problems can be solved by considering the equivalent moment using the sum of the masses.

Let \bar{x} be the distance of the centre of mass from a **chosen fixed point** (usually one of the ends) and let x_1, x_2 , etc be the distances of the masses m_1, m_2 , etc from the chosen fixed point.

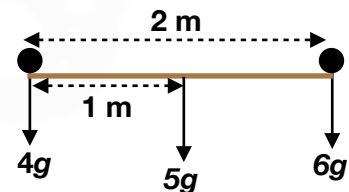
$$(m_1 + m_2 + m_3 + \dots)\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \dots$$

Let M be the sum of the masses then
$$M\bar{x} = \sum_{i=1}^n m_i x_i$$

E.g. 1 A uniform rod of length 2 m has mass 5 kg. Masses of 4 kg and 6 kg are fixed at each end of the rod. Find the centre of mass of the rod.

Working: Let \bar{x} be the distance of the CoM from the 4 kg mass
 ⌚ about 4 kg mass:
 $(4 + 5 + 6)\bar{x} = 6 \times 2 + 5 \times 1$
 $15\bar{x} = 17$
 $\bar{x} = \frac{17}{15} = 1.1\dot{3}$

The centre of mass is 1.13 m from the 4 kg mass



E.g. 2 A rod, AB , of mass 1.1 kg and length 1.2 m has its centre of mass 0.48 m from the end A . What mass should be attached to the end B to ensure that the centre of mass is at the midpoint of the rod?

2-dimensional (i.e. uniform laminas)

In 2-dimensions, the centre of mass $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$ relative to a fixed point (usually the origin) of a series of

masses m_1, m_2 , etc whose centre of masses are at $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ etc. is given by:

$$M \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = m_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + m_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \dots + m_n \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

E.g. 3 Masses of $m, 2m$ and $3m$ are placed at the points $(4, 3), (1, 1)$ and $(3, 0)$. Find the centre of mass of this system.

Working: \curvearrowright about the origin:

$$\begin{aligned} 6m \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} &= m \begin{pmatrix} 4 \\ 3 \end{pmatrix} + 2m \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3m \begin{pmatrix} 3 \\ 0 \end{pmatrix} \\ 6 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} &= \begin{pmatrix} 15 \\ 5 \end{pmatrix} \\ \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} &= \begin{pmatrix} 2.5 \\ \frac{5}{6} \end{pmatrix} \end{aligned}$$

E.g. 4 Find the centre of mass of a body comprising of a uniform square plate of mass 3 kg and side length 2 m with small objects of mass 1 kg, 2 kg, 4 kg and 5 kg at the corners of the square. Assume the 1 kg is at the origin with the other masses placed anti-clockwise around the square.

Video: [CoM \(particles in a line\)](#)

Video: [CoM \(particles in a plane\)](#)

Video (password needed): [Introduction to centres of mass](#)

[Solutions to Starter and E.g.s](#)

Exercise

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Summary

1-dimensional (i.e. rods)

$(m_1 + m_2 + m_3 + \dots)\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \dots$ where m_i are the masses and x_i are the distances from of the masses to the chosen fixed point.

Let M be the sum of the masses then $M\bar{x} = \sum_{i=1}^n m_i x_i$

2-dimensional (i.e. uniform laminas)

$$M \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = m_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + m_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \dots + m_n \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$