Continuous random variables

Starter

1. **(Review of last lesson)**
   A Wilcoxon signed-rank test is carried out on 48 data values and the sum of the positive ranks is 795. Conduct a two-tailed test at the $5\ \%$ level to determine whether the median is different to a previous value. State the null and alternative hypotheses clearly.

Notes

Discrete random variables (drv) were covered at AS level and included the binomial, geometric and Poisson distributions. Discrete random variable can only take specific values distributions and it made sense to calculate probabilities such as $P(X = 3)$ or $P(X \leq 5)$.

One ***continuous random variable (crv)*** has already been met in A2 mathematics with the Normal distribution, where there are no gaps between the values that the variable can take. As such it does not make sense to calculate $P(X = 3)$ so instead probabilities are calculated over a range.

CRVs are defined by functions, often denoted by $f(x)$, called ***probability density functions (pdf)***. In order to calculate probabilities, an integration needs to be performed:

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

Since the sum of the probabilities must add up to one, $\int_{-\infty}^{\infty} f(x)dx = 1$ and since probabilities are positive $f(x) \geq 0$.

N.B. You can use your calculator to do the definite integration calculations unless it specifically states "Show all your working."

E.g. 1 The probability density function, $f(x)$, is defined as $f(x) = \begin{cases} kx^2 & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$.

(a) Find the value of the constant k.
(b) Find $P(X \le 2)$.
(c) Find $P(1.5 \le X \le 2.5)$
(d) Given that $P(X \le a) = 0.2$, find the value of a to 3 s.f.

Working: (a) $\int_{-\infty}^{\infty} f(x)dx = 1$: $\int_0^3 kx^2 dx = 1$

$$\left[\frac{k}{3}x^3\right]_0^3 = 1 \quad \Rightarrow \quad 9k = 1 \quad \Rightarrow \quad k = \frac{1}{9}$$

(b) $$P(X \le 2) = \int_0^2 \frac{1}{9}x^2 dx = \left[\frac{1}{27}x^3\right]_0^2 = \frac{8}{27}$$

(c) $$P(1.5 \le X \le 2.5) = \int_{1.5}^{2.5} \frac{1}{9}x^2 dx = \left[\frac{1}{27}x^3\right]_{1.5}^{2.5} = \frac{49}{108}$$

(d) $$P(X \le a) = 0.2 \quad \Rightarrow \quad \int_0^a \frac{1}{9}x^2 dx = 0.2$$

i.e. $$\left[\frac{1}{27}x^3\right]_0^a = 0.2 \quad \Rightarrow \quad a^3 = 5.4$$

$a = \sqrt[3]{5.4} = 1.75$ (3 s.f.)

E.g. 2 The probability density function $f(x) = \begin{cases} c(4 - x^2) & -2 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

(a) Calculate the value of the constant c.
(b) Find $P(X \ge 0.5)$.
(c) $P(|X| > 1)$.

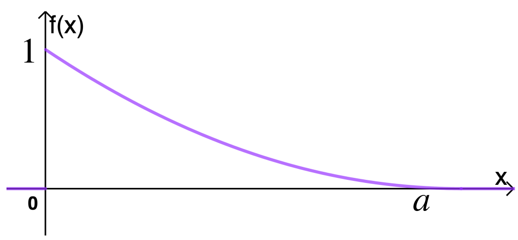
E.g. 3 The crv, X, has pdf, $f(x)$ where $f(x) = \begin{cases} k & 0 \le x \le 2 \\ k(2x - 3) & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$

(a) Find the value of k.
(b) Sketch the graph of $y = f(x)$.
(c) Find $P(1 \le X \le 2.3)$

E.g. 4 The probability density function

$f(x) = \begin{cases} k(x - a)^2 & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$

is shown in the sketch.



(a) Use the information given in the sketch and the properties of probability density functions to find the values of a and k.

(b) Find $P\left(X \ge \frac{1}{2}a\right)$.

Video: Continuous random variables
Video: Continuous random variables (exam questions)

Solutions to Starter and E.g.s

Exercise
p121 7A Qu 1i, 2i, 3i, 4-8, (9-10 red)

Summary
CRVs are defined by functions, often denoted by $f(x)$, called ***probability density functions (pdf)***.
In order to calculate probabilities, an integration needs to be performed:

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