Continuous uniform distribution

Starter

1. (Review of last lesson) The probability density function of a continuous random variable, X, is $f(x) = \begin{cases} \frac{2}{3} & 0 \le x < 1 \\ \frac{4}{3} - \frac{2}{3}x & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$.

(a) Find $F(x)$.

(b) Find the exact value of the upper quartile.

2. A continuous uniform random variable has probability density function $f(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$, where a and b are constants.

(a) Sketch the graph of $f(x)$.

(b) Find the value of k in terms of a and b.

(c) Hence find $P(x_1 \le x \le x_2)$ where $a \le x_1 \le b$ and $a \le x_2 \le b$.

(d) Find the cumulative distribution function, $F(x)$.

Notes

A ***continuous uniform*** (or ***rectangular***) ***distribution*** is the continuous version of the discrete uniform distribution i.e. the probability is the same for each value of x.

A continuous random variable with pdf $f(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$ is sometimes written as $X \sim \text{R}(a, b)$.

E.g. 1 A continuous random variable has pdf $f(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$. Find, in terms of a and b:

(a) E(X).

(b) Var(X).

Working: (a) $k = \dfrac{1}{b-a}$

By symmetry, $\text{E}(X) = \dfrac{1}{2}(a+b)$ or the calculation is:

$$\text{E}(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_a^b \frac{1}{b-a}xdx = \left[\frac{1}{2(b-a)}x^2\right]_a^b$$

$$= \frac{1}{2(b-a)}b^2 - \frac{1}{2(b-a)}b^2 = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$$

E.g. 2 The continuous rv X has probability density function $f(x) = \begin{cases} \frac{1}{4} & 1 \le x \le k \\ 0 & \text{otherwise} \end{cases}$. Find:

(a) the value of k,

(b) E(X)

(c) Var(X)

(d) $P(2.1 < X < 3.4)$

E.g. 3 A continuous uniform random variable has probability density function $f(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$, where a and b are constants. Given that the mean equals 1 and the variance equals $\frac{4}{3}$ find:

(a) the values of a and b

(b) $P(X < 0)$

(c) the value of x such that $P(X > x + \sigma) = \frac{1}{4}$ where σ is the standard deviation. Give your answer to 3 s.f..

E.g. 4 The random variable X has distribution $X \sim \text{R}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(a) Calculate the mean of X.

(b) Calculate the exact value of the variance of X.

(c) Determine the cumulative distribution function, $F(x)$.

Video: Continuous uniform (rectangular) distribution

Video: Continuous uniform distribution (exam question)

Solutions to Starter and E.g.s

Exercise

p137 7F Qu 1i, 2i, 3-6, (7 red)

Summary

Continuous random variable has pdf $f(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$ or $X \sim \text{R}(a, b)$

$$k = \frac{1}{b-a}$$

The cdf is $F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$

$$\text{E}(X) = \frac{1}{2}(a+b)$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$