

Expectation and variance of functions of a random variable

Starter

1. (Review of last lesson)

A continuous random variable X , has the probability density function

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the median value of X .
- State the mode.
- Find $E(X)$.
- Find $\text{Var}(X)$.

Notes

The results for **linear transformations** of discrete random variables derived in the AS course also hold for continuous random variables

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

See this [lesson](#) for the proofs.

E.g. 1 The continuous random variable, X , has pdf $f(x) = \begin{cases} 4x^3 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$.

- Find $E(X)$.
- Find $E(10X + 3)$.
- Find $\text{Var}(X)$.
- Find $\text{Var}(4X - 7)$.

Working: (a) $E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 4x^4 dx = \frac{4}{5}$

(b) $E(10X + 3) = 10 \times \frac{4}{5} + 3 = 11$

(c) $\text{Var}(X) = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2 = \int_0^1 4x^5 dx - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$

(d) $\text{Var}(4X - 7) = 4^2 \times \text{Var}(X) = \frac{32}{75}$

E.g. 2 The number of kilograms of metal ore extracted from 10 kg of ore from a certain mine is modelled by a continuous random variable X with probability density function $f(x)$, where

$$f(x) = \begin{cases} \frac{3}{4}x(2-x)^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the mean and variance of X .
- The cost of extracting the metal from 10 kg of ore is £(9x + 0.4). Find the expected cost of extracting the metal from 10 kg of ore.

General transformations

When finding variance the fact that $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$ can be used.

This can be generalised to $E(g(X)) = \int_{-\infty}^{\infty} g(X) f(x) dx$.

E.g. 3 The crv has pdf $f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$. Find $E(2X^2 + 3X + 3)$

Working: $E(g(X)) = \int_{-\infty}^{\infty} g(X) f(x) dx$

$$E(2X^2 + 3X + 3) = \int_0^1 (2x^2 + 3x + 3)3x^2 dx = \frac{129}{20} = 6.45$$

E.g. 4 The crv has pdf $f(x) = \begin{cases} \frac{1}{18}(6-x) & 0 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$. Find $E(X^2 - 4X + 3)$

Video: [Expectation and variance of a continuous random variable](#)

[Solutions to Starter and E.g.s](#)

Exercise

p128 7C Qu 1i, 2i, 3i, 4-7, (8-9 red)

Summary

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(X) f(x) dx$$