Impulse-Momentum of Variable Forces

Starter

1. **(Review of last lesson)**
A rock of mass 10 kg falls over a cliff and drops vertically on to a field 200 m below. The air resistance is given by $0.0392v^2$ N when the speed is v m/s. Find:
(a) the terminal speed for a fall of indefinite distance
(b) the speed with which the rock hits the field

The impulse-momentum principle for constant forces

From AS, impulse equals the increase in momentum.
Impulse-momentum principle:

Increase in speed: $\text{Impulse} = Ft = mv - mu$
Decrease in speed: $\text{Impulse} = Ft = mu - mv$

2. **(Review of AS FM material)**
A particle of mass 3 kg is moving along a straight line in the direction AB with speed 6 m/s when a force is applied to it. After 4 seconds the particle is moving in the direction BA with speed 2 m/s. Find the magnitude and direction of the force.

Notes

The impulse-momentum principle for variable forces

When the force, F, is not constant i.e. depends on t, we can use $F = ma$:

$$F(t) = m\frac{dv}{dt}$$
$$\int_0^t F(t)dt = \int_u^v m\,dv$$
$$\int_0^t F(t)dt = mv - mu$$

Since $mv - mu$ is the change in momentum we know that:

$$\text{Impulse} = \int_0^t F(t)dt$$

E.g. 1 A body of mass 1.4 kg falls from rest in a medium which exerts a resistance of $(2t + k)$ N, where k is a constant. The speed of the body after falling for 4 seconds is 18 m/s. Find:
(a) the value of k
(b) the speed after a further 3 seconds.

Working (a) Resultant force, $F = 1.4g - (2t + k) = 13.72 - 2t - k$
When $t = 0$, $v = 0$ and when $t = 4$, $v = 18$ so

$$\text{Impulse} = \int_0^4 (13.72 - 2t - k)dt = 1.4 \times 18 - 1.4 \times 0$$
$$\int_0^4 (13.72 - 2t - k)dt = 25.2$$
$$\left[13.72t - t^2 - kt\right]_0^4 = 25.2$$
$$54.88 - 16 - 4k = 25.2$$
$$k = 3.42$$

Momentum and vector notation

The same principles and equations that we learnt in AS also apply when quantities are given in vector form.

Impulse: $\mathbf{F}t = m\mathbf{v} - m\mathbf{u}$

Conservation of momentum: $m_1\mathbf{u_1} + m_2\mathbf{u_2} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$

E.g. 2 A body of mass 2 kg moving with velocity $(3\mathbf{i} + 4\mathbf{j})$ m/s, collides with a body of mass 3 kg, moving with velocity $(-4\mathbf{i} + 5\mathbf{j})$ m/s. After the collision the two bodies coalesce. Find the common velocity of the combined body after the impact.

E.g. 3 A particle, A, of mass 500 g, is acted on by a variable force F N, which is defined as:

$F = 0.3t^2 + 0.5t$ for $0 \leq t \leq 3$ s

$F = t + 1.2$ for $3 < t \leq 5$ s.

Find the speed of A after 4 seconds if the initial speed is 2 m/s.

Video (password needed): Work done by a variable force

Solutions to Starter and E.g.s

Exercise

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Summary

For a constant force, F: Impulse $= Ft = mv - mu$

For a variable force, $F(t)$: $\text{Impulse} = \int_{t_1}^{t_2} F(t)dt$