Integration of hyperbolic functions

Starter

1. **(Review of last lesson)**
Calculate the $y-$value of the stationary point of the curve $y = 25\cosh x - 7\sinh x$.

2. Using the definition of $\sinh x$, prove that $\int \sinh x\,dx = \cosh x + c$.

3. State: (a) $\int \cosh x\,dx$ (b) $\int \tanh x\,dx$

Notes

Here are the standard integrals of hyperbolic functions:

$$\int \sinh x\,dx = \cosh x + c$$
$$\int \cosh x\,dx = \sinh x + c$$
$$\int \tanh x\,dx = \ln\cosh x + c$$

All the integration methods learnt apply with hyperbolic functions.

Integrating hyperbolic functions is easier than trigonometric functions because when in doubt one can always fall back on the definitions in terms of e^x.

E.g. 1 Find the following integrals:

(a) $\int \sinh 5x\,dx$ (b) $\int x\sinh x\,dx$

(c) $\int 6\sinh^2 x\,dx$ (d) $\int e^x\cosh x\,dx$

Working: (a) $\int \sinh 5x\,dx = \frac{1}{5}\cosh 5x + c$

E.g. 2 Find the exact value of $\int_0^2 8e^x\sinh 2x\,dx$.

E.g. 3 Find the exact area between the curve of $y = 5 - \cosh 4x$ and the $x-$axis, giving your answer in the form $\frac{a}{b}\ln(5+2\sqrt{6}) - \sqrt{c}$ where a, b and c are integers to be found.

Video: Integrating expressions involving hyperbolic functions

Solutions to Starter and E.g.s

Exercise

p146 6F Qu 1i, 2i, 3i, 4i, 5i, 6-12 (not 8, 11)

Summary

$$\int \sinh x\,dx = \cosh x + c$$

$$\int \cosh x\,dx = \sinh x + c$$

$$\int \tanh x\,dx = \ln \cosh x + c$$

When integrating functions involving $\sinh$ and $\cosh$, it can be easier to use the definitions in terms of e^x