Linear combinations of normal variables

Starter

1. **(Review of last lesson)** Find the best estimates of the population mean and standard deviation from this sample. Give your answers to 3 s.f..

Weight	$50 \leq x < 55$	$55 \leq x < 60$	$60 \leq x < 70$	$70 \leq x < 80$	$80 \leq x < 85$
Frequency	23	28	61	54	19

2. Given that $X \sim \mathrm{N}(12, 1.4)$, find the value of:
   (a) $P(X > 13)$
   (b) $P(X \leq 10.6)$
   (c) $P(11.2 \leq X \leq 12.5)$
   (d) a such that $P(X \geq a) = 0.63$

Using a calculator

Probabilities Menu >> 7:Distribution >> 2:Normal CD

Input screen **Normal CD**

Lower: **type a large number like -99999 if $P(X < x)$**
Upper: **type a large number like 99999 if $P(X > x)$**
σ: **square root to get standard deviation if $X \sim \mathrm{N}(5, 7)$**
μ: ***you may need to scroll down to see this***

$x-$***values*** Menu >> 7:Distribution >> 3:Inverse Normal

Input screen **Inverse normal**

Area: **type a large number between 0 and 1**
σ: **square root to get standard deviation if $X \sim \mathrm{N}(5, 7)$**
μ:

Video (Classwiz): **Finding probabilities for the normal distribution**

Notes

A linear combination of random variables that are normally distributed is also normally distributed.

So if $X \sim \mathrm{N}(\mu_x, \sigma_x^2)$ and $Y \sim \mathrm{N}(\mu_y, \sigma_y^2)$ then:

$$aX + bY \sim \mathrm{N}(a\mu_x + b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$$
$$aX - bY \sim \mathrm{N}(a\mu_x - b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$$

This follows from a previous lesson whereby:

$E(aX \pm bY \pm c) = aE(X) \pm bE(Y) \pm c$ — X and Y are any two random variables
$\mathrm{Var}(aX \pm bY \pm c) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$ — X and Y are independent random variables

E.g. 1 Let X and Y be independent random variables such that $X \sim \mathrm{N}(48, 25)$ and $Y \sim \mathrm{N}(185, 144)$. Find:
(a) $P(X + Y > 230)$
(b) $P(Y - X \leq 125)$
(c) $P(238 < 2Y - 3X \leq 245)$
(d) the value of k such that $P(5X - Y \leq k) = 0.79$

Working: (a) $X + Y \sim \mathrm{N}(48 + 185, 25 + 144) = \mathrm{N}(233, 13^2)$
$P(X + Y > 230) = 0.591$ (3 s.f.)

E.g. 2 Each Tuesday, Sally leaves home to play tennis. The time she takes to travel to and from the tennis courts is normal distributed with mean of 50 minutes and standard deviation of 10 minutes. The length of her match is also a normal variable with mean 62 minutes and standard deviation 13 minutes. Find the probability that:

(a) Sally is away from home for more than 2 hours

(b) Sally spends more time travelling than playing tennis.

Give your answers to 3 s.f..

Remember $E(\bar{X}) = E(X)$ and $\text{Var}(\bar{X}) = \dfrac{\text{Var}(X)}{n}$.

E.g. 3 Let X be a random variable such that $X \sim \text{N}(35, 4^2)$. If 3 independent observations of X are made, find $P(\bar{X} > 34)$ to 3 s.f..

Working:

$$E(\bar{X}) = 35$$

$$\text{Var}(\bar{X}) = \frac{\text{Var}(X)}{n} = \frac{4^2}{3} = \frac{16}{3}$$

$$\bar{X} \sim \text{N}\left(35, \frac{16}{3}\right)$$

$$P(\bar{X} > 34) \approx 0.667497 = 0.667 \text{ (3 s.f.)}$$

E.g. 4 Let Y be a random variable such that $Y \sim \text{N}(290, 108)$. If 9 independent observations of Y are made, find $P(\bar{Y} \leq 295)$ to 3 s.f..

Video: Linear combinations of normal distributions

Solutions to Starter and E.g.s

Exercise

p159 8D Qu 1i, 2-8 (9-11 red)

Summary

If $X \sim \text{N}(\mu_x, \sigma_x^2)$ and $Y \sim \text{N}(\mu_y, \sigma_y^2)$ then:

$$aX + bY \sim \text{N}(a\mu_x + b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$$
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