Oblique collisions of two objects

Starter

1. **(Review of last lesson)** A smooth sphere of mass 5 kg strikes a wall with speed 10 m/s at an angle of 60^o to the wall. The impulse on the sphere is 60 Ns. Find, in any order:
   (a) the rebound speed
   (b) the angle of rebound
   (c) the coefficient of restitution.

2. **(Review of AS FM material)**
   In a game of snooker the white ball, moving at 2 m/s, strikes the pink full on. The pink starts to move with a speed of 1.6 m/s. Given that the balls have the same mass, find:
   (a) the speed of the white ball after impact and,
   (b) the coefficient of restitution, e.

Notes

When a collision occurs between two smooth spheres:

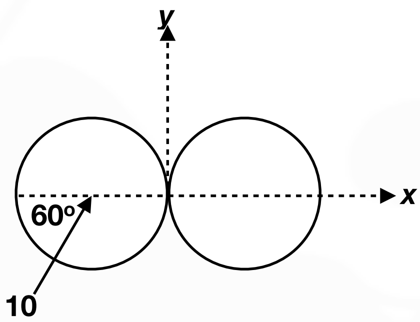
- The ***impulse*** from each sphere ***acts along the line of centres*** (through the point of contact).
- The component of ***velocity*** of each sphere ***perpendicular to the line of centre remains the same***.
- The ***component of velocity along the line of centres*** follows ***Newton's law of impact*** and the ***conservation of momentum***.

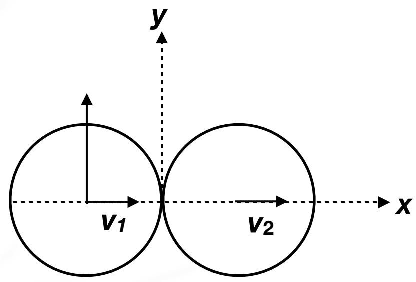
When solving problems
Along the line of centres — use ***conservation of momentum*** and ***Newton's law of impact***
Perpendicular to the line of centres — ***components of velocity remain the same***.

E.g. 1 In a game of shove-ha'penny, a coin is moving along the board at a speed of 10 m/s when it clips a second coin, of the same mass, at an angle. At the instant when the coins come into contact, the first coin is moving at 60° to the line of centres. The coefficient of restitution is 0.6. Find how the two coins are moving immediately after the collision.
Hint: draw the line of centres across the page (the line of centres becomes the $x-$axis).

Working: The impulse acts along the line of centre. Therefore, the 2nd coin will move away along the $x-$axis. Let v_1 be the speed component of the 1st coin and let v_2 be the speed of the 2nd coin in the $x-$direction.
The $x-$ and $y-$components of the velocity are calculated separately.

Before the collision **After the collision**



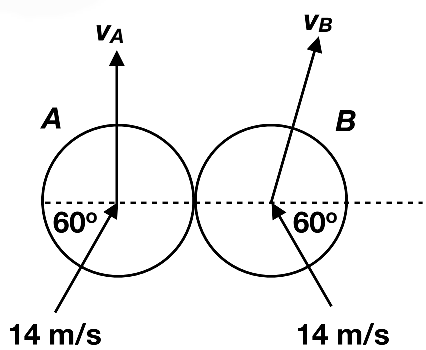


$x-$***direction (parallel to the line of centres):***
Conservation of momentum: $10m\cos 60 = mv_1 + mv_2$
$\therefore v_1 + v_2 = 5$
Newton's law of impact: $v_2 - v_1 = e \times 10\cos 60$
$-v_1 + v_2 = 3$
Solving simultaneously gives: $v_1 = 1$ $v_2 = 3$

$y-$***direction (perpendicular to the line of centres):***
velocity component before = velocity component after
1st coin: $10\sin 60 = 5\sqrt{3}$
2nd coin: 0

After the collision, the velocities of the coins are $\begin{pmatrix} 1 \\ 5\sqrt{3} \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

E.g. 2 Two smooth spheres, A and B, of equal radius, have the same speed of 14 m/s immediately before they collide. The mass of A is 0.8 kg and the mass of B is 0.7 kg. Before the collision the path of each sphere makes an angle of 60° with the line of centres, and immediately after the collision A moves perpendicular to the line of centres (see diagram).
Calculate:
(a) the speed of A immediately after the collision
(b) the coefficient of restitution.



In vector form

When velocities are given in term of vectors, make sure you take note which direction the line of centre is.

E.g. 3 Two smooth spheres, A and B, with equal radii, lie on a horizontal plane. The mass of B is twice that of A. The spheres are projected towards each other and they collide when the line joining their centres is in the direction of the unit vector $\mathbf{i}$. The velocity vectors of A and B just before impact are $(2\mathbf{i}+\mathbf{j})$ m/s and $(\mathbf{i}-\mathbf{j})$ m/s respectively. If $e=\frac{1}{2}$, find:

(a) their velocity vectors just after impact
(b) the kinetic energy lost due to the collision

Working: (a) Since the line of centres is in the $\mathbf{i}-$direction, the velocity components in the $\mathbf{j}-$direction are unchanged.
Let the mass of A be m so the mass of B is $2m$.
Let the velocities of A and B in the $\mathbf{i}-$direction after impact be v_A and v_B respectively.

Conservation of momentum:

$$2m + 2m = mv_A + 2mv_B$$
$$v_A + 2v_B = 4$$

Newton's law of impact:

$$v_B - v_A = \frac{1}{2}(2-1)$$
$$-2v_A + 2v_B = 1$$

Solving simultaneously: $v_A = 1$ $\quad v_B = \frac{3}{2}$

The velocity vectors just after impact are A: $(\mathbf{i}+\mathbf{j})$ m/s and B: $\left(\frac{3}{2}\mathbf{i}-\mathbf{j}\right)$ m/s.

(b) Since the velocity component in the $\mathbf{j}-$direction are unchanged, only the velocity in the $\mathbf{i}-$direction needs to be considered.
KE lost = KE before − KE after

$$= \frac{1}{2}m \times 2^2 + \frac{1}{2} \times 2m \times 1^2 - \frac{1}{2}m \times 1^2 - \frac{1}{2} \times 2m \times \left(\frac{3}{2}\right)^2$$
$$= 2m + m - \frac{1}{2}m - \frac{9}{4}m$$
$$= \frac{1}{4}m$$

The kinetic energy lost due to the collision is $\frac{1}{4}m$

Video (password needed): Oblique impact with a stationary sphere
Video (password needed): Oblique impact with a moving sphere
Video (password needed): Loss of kinetic energy and deflection in oblique impacts

Solutions to Starter and E.g.s

Exercise
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Summary

- The ***impulse*** from each sphere ***acts along the line of centres*** (through the point of contact).
- The component of ***velocity*** of each sphere ***perpendicular to the line of centre remains the same***.
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