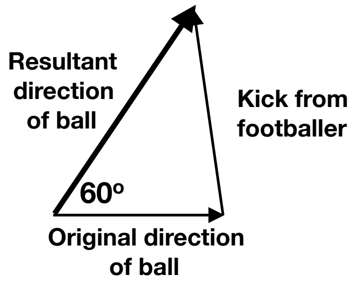
Oblique impacts with walls

Starter

1. **(Review of last lesson)** Two balls, A and B, of masses 1.2 kg and 0.8 kg respectively, are rolling on a smooth horizontal floor when they collide. Initially their velocities are $(4.2\mathbf{i} + 2.6\mathbf{j})$ m/s and $(1.2\mathbf{i} - 0.4\mathbf{j})$ m/s respectively. After the collision the velocity of A is $(1.4\mathbf{i} + 1.8\mathbf{j})$ m/s. Find the velocity of B after the collision.

2. When a footballer receives the ball it is moving at 8 m/s. She kicks it so that its direction is diverted through 60^o and its speed is increased to 20 m/s. In what direction does the player kick the ball?

   Hint: Use the diagram to help you.



Notes

From AS:

Newton's law of impact: $e \times$ approach $=$ separation

$e(u_1 - u_2) = v_2 - v_1$ or $v_2 - v_1 = -e(u_2 - u_1)$

For an object hitting a wall at ***right angles***: $v = -eu$

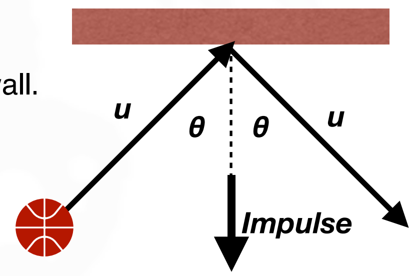
Oblique impacts means impacts that do not happen at right angles to the wall.
In questions, motion will be restricted to:

1. Uniform objects with spherical or circular symmetry, such as balls or discs
2. Smooth surfaces
3. Objects that do not spin

***Coefficient of restitution,* $\mathbf{e} = 1$**

If a ball is kicked at a smooth wall whose coefficient of restitution, e, is 1, the ball will bounce back at the same angle at which it hit the wall.

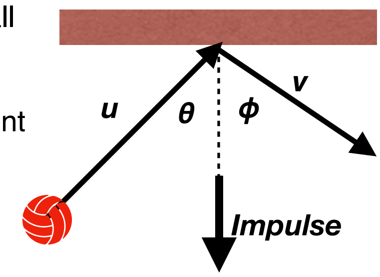
N.B. The impulse is perpendicular to the wall.



***Coefficient of restitution,* $\mathbf{e} \neq 1$**

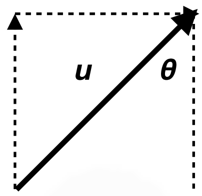
If the wall is smooth but the coefficient of restitution, e, is ***not*** 1, the ball will bounce back at a different angle. Its speed will also change.

N.B. The impulse is again perpendicular to the wall so the component of the velocity parallel to the wall will remain unchanged.
The velocity perpendicular to the wall will change.

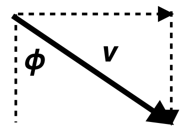


E.g. 1 Copy the diagrams and write down the components of velocity parallel and perpendicular to the wall.

Before impact



After impact



Working:

	Before impact	After impact
Parallel to wall:	$u\sin\theta$	$v\sin\phi$
Perpendicular to the wall:	$u\cos\theta$	$-v\cos\phi$

Impulse is perpendicular to the wall so e does not affect the speed parallel to the wall

Since the ***speed parallel to the wall is unchanged***: $v\sin\phi = u\sin\theta$

Remember, if motion is perpendicular to the wall $v = -\,eu$

Perpendicular to the wall we use e: $v\cos\phi = -\,eu\cos\theta$

To find the final velocity: $v = \sqrt{v_{//}^2 + v_{\perp}^2}$

When solving problems, consider the velocity

- ***Parallel, //, to the plane*** the object hits (speed remains the same)
- ***Perpendicular, ⊥, to the plane*** the object hits.

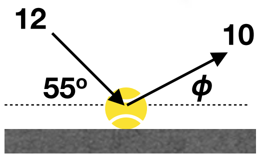
E.g. 2 A smooth sphere of mass m strikes a fixed plane surface with speed 12 m/s at an angle 55^{o} to the plane and rebounds with speed 10 m/s. Find:

(a) the angle at which it rebounds

(b) the coefficient of restitution

Working (a) Speed // to plane remains the same:

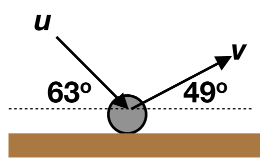
$$10\cos\phi = 12\cos 55$$
$$\phi = 46.5^{\text{o}} \text{ (3 sf)}$$



(b) Speed ⊥ to the plane (use e):

$$e \times 12\sin 55 = 10\sin 46.5$$
$$e = 0.738 \text{ (3 s.f.)}$$

E.g. 3 In ice-hockey the playing area is bounded by a vertical wooden barrier. A puck strikes the barrier at an angle of 63^{o} and rebounds at an angle of 49^{o}. Calculate the coefficient of restitution.

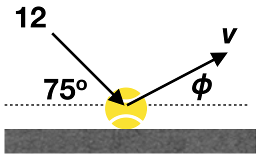


Finding the impulse

Impulse = change in momentum = final momentum — initial momentum

Since impulse acts perpendicular to the plane to contact, we can just consider ***the velocity perpendicular to the wall***.

E.g. 4 A ball of mass 0.2 kg moving at 12 m/s hits a smooth horizontal wall at an angle of 75^o to the horizontal. The coefficient of restitution is 0.5. Find:



(a) the speed of the ball, v, as it leaves the wall
(b) the impulse on the ball
(c) the impulse on the plane
(d) the kinetic energy lost by the ball.

Working (a) Speed // to the plane remains the same:

Velocity $= 12\cos 75$ (no change)

Speed $\perp$ to the plane (use e):

Velocity $= 0.5 \times 12\sin 75 = 6\sin 75$

$$v = \sqrt{(12\cos 75)^2 + (6\sin 75)^2} = 6.58 \text{ m/s (3 s.f.)}$$

(b) Impulse is the change in momentum ***perpendicular*** to the wall.

Impulse = final momentum — initial momentum

$$I = 0.2 \times 6\sin 75 - 0.2 \times (-12\sin 75)$$

$I = 3.6\sin 75 = 3.48$ Ns perpendicular to the wall

(c) By Newton's 3rd law, impulse on the plane $= 3.48$ Ns (3 s.f.)

(d) KE lost $= \frac{1}{2} \times 0.2 \times 12^2 - \frac{1}{2} \times 0.2 \times 6.58^2 = 10.1$ J (3 s.f.)

E.g. 5 A ball of mass 0.1 kg moving at 10 m/s hits a smooth horizontal plane at an angle of 80^o to the horizontal. The coefficient of restitution is 0.6. Find:

(a) the impulse on the ball
(b) the kinetic energy lost by the particle

Video (password needed): Oblique impact with a fixed surface
Video (password needed): Loss of kinetic energy in an oblique impact
Video (password needed): Successive oblique impacts with plane surfaces

Solutions to Starter and E.g.s

Exercise
p210 8B Qu 1-4 (Red 5-11)

Summary
$\mathbf{e = 1}$ — the ball will bounce back at the same angle at which it hit the wall,
$\mathbf{e \neq 1}$ — the ball will bounce back at a different angle and its speed will also change.

- Impulse is perpendicular to the wall
- The speed parallel to the wall will remain unchanged.
- The speed perpendicular to the wall will change.

When solving problems, consider the speed:

- ***Parallel to the plane*** the object hits (speed remains the same).
- ***Perpendicular to the plane*** the object hits (use e).

Finding the impulse:
Since impulse acts perpendicular to the plane to contact, we can just consider ***the velocity perpendicular to the wall***.