Review of proof by induction

Starter

1. **(Review of last lesson)**
   Find the area enclosed between the curves $C_1 : \ r = 3 + \cos\theta$ and $C_2 : r = 7\cos\theta$.

2. **(Review of previous material)**
   Given that $\mathbf{A} = \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}$, prove by induction that $\mathbf{A}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 1+3n \end{pmatrix}$.

Notes

As covered in AS further maths, there are four stages to proof by induction:

1. ***(Proposition)*** Statement of the proposition $P(n)$
2. ***(Prove the basic case)*** Prove the result for $n = 1$.
3. ***(Inductive steps)*** Assume the result is true for k and then prove that if the result for k is true then the result is also true for $k + 1$
4. ***(Completion of the proof)*** "But this is $P(k)$ with k replaced by $k + 1$. Therefore, if $P(k)$ is true, then $P(k + 1)$ is also true. $P(1)$ is true and if $P(k)$ is true, then $P(k + 1)$ is true. Using the principle of mathematical induction, $P(n)$ is true for all positive integers."

The key step is the inductive step. The final line of the algebraic manipulation must be the same formula as $P(k)$ but with k replaced by $k + 1$. It is a good idea to write down in pencil in the margin what it is that you are aiming for.

The types of question covered at AS were: matrices, divisibility, n-th term of a sequence using subscript notation and inequalities.

E.g. 1 Prove that $u_n = 4^n + 6n - 1$ is divisible by 9 for all $n \geq 1$ where n is an integer.

E.g. 2 Prove that $3^n > n^2$ for all integers $n > 2$.

Video: Proof by induction (matrices)
Video: Proof by induction (divisibility)
Video A: Proof by induction (inequalities)
Video B: Proof by induction (inequalities)

Solutions to Starter and E.g.s

Exercise

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Summary

There are four stages to proof by induction:

1. ***(Proposition)*** Statement of the proposition $P(n)$
2. ***(Prove the basic case)*** Prove the result for $n = 1$.
3. ***(Inductive steps)*** Assume the result is true for k and then prove that if the result for k is true then the result is also true for $k + 1$
4. ***(Completion of the proof)*** "But this is $P(k)$ with k replaced by $k + 1$. Therefore, if $P(k)$ is true, then $P(k + 1)$ is also true. $P(1)$ is true and if $P(k)$ is true, then $P(k + 1)$ is true. Using the principle of mathematical induction, $P(n)$ is true for all positive integers."