The single-sample sign test

Notes

What is a parameter?

A parameter is a number that defines a system or set of conditions

Examples of parameters

- n and p in the binomial distribution
- μ in a normal distribution

Non parametric tests

Non-parametric tests are hypothesis tests that do not assume that the population follows a particular distribution. They are sometimes also called ***distribution-free tests***. They allow a hypothesis test to be carried out on a ***small sample*** when it is not certain what the distribution of the population is.

Non-parametric tests are not very accurate or very powerful (i.e. cannot detect a difference when there is one) but they are robust (I.e. still valid even if some of the assumptions are a little out).

Single-sample sign test

The ***single-sample sign test*** looks at a set of data and tries to determine whether the median has changed from a previous known, or assumed, value.

The null and alternative hypotheses are:

H_0 : Median $=$ the known value

H_1 : Median is not this known value (either "not equal to", "greater than" or "less than")

Often with single-sample sign tests the sample is small so we have little information about the distribution. With this test ***no assumptions*** are made about the distribution.

If a data value from the sample is ***equal to the median***, it is ***simply ignored*** (although this situation does not appear in the OCR specification). Therefore, any value from the sample is either above the median or below the median.

Let X be the number of values above (or below) the median as given by the null hypothesis. If n is the size of the sample, we expect the data to be either side of the median, following a binomial distribution, $X \sim \text{B}(n,\ 0.5)$.

The test statistic, x, is the number of values from the sample which are above or below the median. The test is carried out at $\alpha\ \%$ significance level if above or below, or $\frac{\alpha}{2}\ \%$ if is is a two-tail test.

If $p = P(X \geq x) \nleq \alpha\ \%$ (one-tailed) or $p = P(X \geq x) \nleq \frac{\alpha}{2}\ \%$ (two-tailed), we do not reject H_0, i.e. there is no evidence to suggest the median has changed.

If $p = P(X \geq x) \leq \alpha\ \%$ (one-tailed) or $p = P(X \geq x) \leq \frac{\alpha}{2}\ \%$ (two-tailed), we reject H_0, and state that there is evidence to suggest the median has changed.

N.B. $P(X \geq x)$ or $P(X \leq x)$ is called the $p-$value.

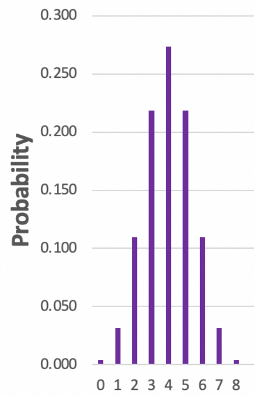
Since the binomial distribution is symmetrical, i.e. ${}^nC_r = {}^nC_{n-r}$, so
$p = P(X \geq x) = P(X \leq n - x)$

E.g. Leona plays golf where the aim is to take as few shots as possible. Her median number of shots for a round is 85. After some lessons from the golf pro, she plays eight rounds in the following number of shots:

$$82 \quad 78 \quad 89 \quad 84 \quad 74 \quad 91 \quad 80 \quad 77$$

(a) Test at the $10\,\%$ level to see if Leona has improved her game.

(b) If in (a) H_0 is not rejected, state how many rounds would have had to be below 85 in order for the the test to show significance.



Working: (a) H_0 : Median is 85
H_1 : Median is less than 85
Let X be the number of values below 85
so $X \sim$ B$(8, 0.5)$
The signs of deviations from the median are:
$- \quad - \quad + \quad - \quad - \quad + \quad - \quad -$
$p = P(X \geq 6) = P(X \leq 2) = 0.145$
Since $p = 0.145 \nleq 0.1$, we do not reject H_0.
There is evidence to suggest that Leona has not improved as a golfer.

(b) $P(X \geq 7) = P(X \leq 1) = 0.0352 \leq 0.1$
So if 7 of Leona's rounds of golf were below 85, there would have been evidence at the $10\,\%$ (and, in fact, the $5\,\%$ level) to suggest she had improved.
N.B. $X \geq 7$ is the critical region of the test.

In fact, the median could be anywhere between 85 and 88 inclusive and the conclusions would be the same. This is an example of what makes this kind of test ***robust***.

Success criteria — single-sample sign test

1. State the H_0 : and H_1 : clearly.
2. State the binomial distribution according to the given median
3. Count the number of signs of deviations, x, from the median.
4. Use a calculator to find $p = P(X \geq x) = P(X \leq n - x)$ where n is the number of data values.
5. Compare p to the level of significance ($\alpha\,\%$ if one-tailed, $\frac{\alpha}{2}\,\%$ if two-tailed)
6. If $p = P(X \geq x) \leq \alpha\,\%$, reject H_0 : etc…

E.g. 1 An airline ran a course for ten people who have a fear of flying. They were asked to rate the course on a scale of 1 (very dissatisfied) to 10 (very satisfied). The results were:

$$1 \quad 2 \quad 4 \quad 8 \quad 8 \quad 8 \quad 9 \quad 9 \quad 10 \quad 10$$

(a) Use a single-sample sign test to test, at the $10\,\%$ significance level, the null hypothesis $M = 6$, against the alternative hypothesis $M > 6$, where M is the median rating given for such courses.

(b) What is the minimum number of values that need to be higher than 6 so that the null hypothesis is rejected at the $5\,\%$

E.g. 2 The times, in minutes, spent travelling to school by the 15 members of a statistics class are as follows:

$13 \quad 23 \quad 15 \quad 21 \quad 22 \quad 18 \quad 30 \quad 25 \quad 45 \quad 12 \quad 17 \quad 24 \quad 32 \quad 28 \quad 29$

The school prospectus claims that 'half of the pupils live less than 20 minutes from the school'.

(a) Use a one-tailed sign test, at the $5\,\%$ level, to test whether the median is higher than that stated by the school.

(b) Find the critical region of this test.

E.g. 3 A paper company plans to buy a wood from a farmer who claims that the trees have an average of 15 cm³ of usable wood. The company employs a statistician to check this claim, as they believe the median is less than 15 cm³. With agreement of the farmer the statistician has a random sample of 13 trees felled and their usable wood collected. Here are the results:

$11.3 \quad 8.8 \quad 6.4 \quad 8.7 \quad 14.6 \quad 16.2 \quad 12.4 \quad 7.1 \quad 15.2 \quad 10.3 \quad 9.5 \quad 17.2 \quad 7.8$

What is the researcher's conclusion after conducting a test at the $5\,\%$ significance level?

If data values equal the median, they are simply ignored and the sample size is consequently reduced (remember, this is not in the OCR syllabus).

E.g. 4* Applicants to university may have to wait before being invited to interviews. One university claims that the median waiting time for its applicants is 31 days. A random sample of 18 successful applicants waited for these times:

$33 \quad 32 \quad 35 \quad 30 \quad 33 \quad 37 \quad 30 \quad 31 \quad 37$
$29 \quad 31 \quad 31 \quad 32 \quad 34 \quad 30 \quad 31 \quad 33 \quad 38$

Test, at the $10\,\%$ level, the claim that the median is 31 days against the alternative that it is not 31 days.

Video (password needed): Introduction to non-parametric tests
Video (password needed): The single-sample sign test
Video (password needed): Sign test with a single sample

Solutions to Starter and E.g.s

Exercise
p49 4A Qu 1-5, (6 red)

Summary
The ***single-sample sign test*** looks at a set of data and tries to determine whether the median has changed from a previous known, or assumed, value.

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With this test, ***no assumptions*** are made about the distribution.

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5. Compare p to the level of significance ($\alpha\,\%$ if one-tailed, $\frac{\alpha}{2}\,\%$ if two-tailed)
6. If $p = P(X \geq x) \leq \alpha\,\%$, reject H_0 : etc…