Unbiased estimates of the mean and the variance

Starter

1. **(Review of last lesson)** A company produced bricks with a mean mass 3.1 kg and standard deviation 85 g. They are transported in crates weighing 92 kg. Given that a full crate can hold 480 bricks, calculate the mean and standard deviation of a crate of bricks.

Notes

After collecting a random sample of a population with unknown mean, μ, and unknown variance, σ^2, are the mean and variance of the sample ***unbiased estimates*** (i.e. best) for the population?

For the mean "yes" i.e. the mean of the sample is the best estimate for the mean of the population.

However, it can be shown that the variance of a sample is not an ***unbiased estimate*** for the population variance. In fact, the values given by samples tend to ***underestimate*** that of the population. Therefore, the sample variance is multiplied by $\frac{n}{n-1}$ to get the ***unbiased estimate***.

Notation

$\sigma^2 \equiv$ population variance

$s^2 \equiv$ unbiased estimate of the population variance from a sample

$$s^2 = \frac{n}{n-1} \times \text{sample variance}$$

Since variance is calculated using the formula $\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$:

$$s^2 = \frac{n}{n-1}\left(\frac{\sum x_i^2}{n} - \bar{x}^2\right)$$

E.g. 1 A sample of 16 items is taken from a population such that $\sum x_i = 173$ and $\sum x_i^2 = 2894$. Find unbiased estimates of the population mean and variance.

Working:

$$\bar{x} = \frac{\sum x_i}{n} = \frac{173}{16} = 10.8125$$

$$s^2 = \frac{n}{n-1}\left(\frac{\sum x_i^2}{n} - \bar{x}^2\right) = \frac{16}{15}\left(\frac{2894}{16} - \left(\frac{173}{16}\right)^2\right) = \frac{3275}{48} \approx 68.2$$

Unbiased estimates of the population mean and variance are 10.8125 and 68.2 (3 s.f.) respectively.

E.g. 2 Calculate unbiased estimates of the population mean and variance from which this sample is drawn:

$$8.3,\ 7.9,\ 9.5,\ 6.4,\ 7.2,\ 9.1$$

Finding an unbiased estimate for the variance of population using a Classwiz

Menu >> 6: Statistics >> 1: 1-Variable >> (Enter data) >> Press AC >> Press OPTN >> 2: 1-Variable Calc >> (Data displayed)

$\bar{x}$ ≡ mean (whether data entered was the ***population or*** a ***sample***)
$\sigma^2 x$ ≡ population variance (data entered was the ***population***)
σx ≡ standard deviation of the population (data entered was the ***population***)
$s^2 x$ ≡ unbiased estimate for population variance (data entered was a ***sample***)
sx ≡ unbiased estimate for population standard deviation (data entered was a ***sample***)

N.B. Always check your data after entering to make sure all values are correct.

E.g. 3 Use your calculator to find unbiased estimates of the population mean and variance from the sample:

$$24.6,\ 17.5,\ 9.8,\ 13.6,\ 12.7,\ 11.9\ 13.8,\ 22.9,\ 15.4,\ 14.6,\ 12.3,\ 16.1$$

E.g. 4 Find the best estimate of the population mean and standard deviation for the sample:

Height	$150 \le x < 160$	$160 \le x < 170$	$170 \le x < 175$	$175 \le x < 180$	$180 \le x < 190$
Frequency	9	13	8	7	10

Video: Finding an unbiased estimator for the variance

Video: Proof that sample variance is an unbiased estimator of population variance

Solutions to Starter and E.g.s

Exercise

p157 8C Qu 1i, 2-4

Summary

s^2 ≡ unbiased estimate of the population variance from a sample

$$s^2 = \frac{n}{n-1} \times \text{sample variance} \quad \Rightarrow \quad s^2 = \frac{n}{n-1}\left(\frac{\sum x_i^2}{n} - \bar{x}^2\right)$$

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