Variable forces

Starter

1. (Review of last lesson) A vehicle of mass 500 kg is moving at 25 m/s along a straight horizontal road when the engine cuts out. It is slowed down by air resistance of amount $5v^{\frac{3}{2}}$ N, where v is the speed in m/s. How far does it travels in coming to rest?

Notes

When working with a variable force the key is to choose the version of acceleration that allows you to set up a differential equation.

Remember: $a = \dfrac{dv}{dt} = v\dfrac{dv}{dx}$

Force as a function of time

When force is a function of time i.e. $F(t)$, use $a = \dfrac{dv}{dt}$.

From $F = ma$:

$$F(t) = m\frac{dv}{dt}$$
$$\int F(t)dt = \int m\,dv$$
$$mv = \int F(t)dt$$
$$v = \frac{1}{m}\int F(t)dt$$

E.g. 1 The force acting on a particle of mass 1 kg is given by $F(t) = 5t + 1$. Given that the particle is initially at rest at the origin, find the value of v when $t = 2$.

Force as a function of displacement

When force is a function of displacement i.e. $F(x)$, use $a = v\dfrac{dv}{dx}$.

From $F = ma$:

$$F(x) = mv\frac{dv}{dx}$$
$$\int F(x)dx = \int mv\,dv$$
$$\frac{1}{2}mv^2 = \int F(x)dx$$
$$v^2 = \frac{2}{m}\int F(x)dx$$

E.g. 2 The force acting on a particle of mass 1 kg is given by $F(x) = \dfrac{2}{5+x}$. Given that the particle was initially at rest at the origin, find the value of v when $x = 5$.

Force as a function of velocity

When force is a function of velocity i.e. $F(v)$, either of the equations of for acceleration can be used, depending on whether an equation for time, t, or an equation for displacement, x, is needed. Using $F = ma$:

$$F(v) = m\frac{dv}{dt}$$
$$\int dt = \int \frac{m}{F(v)} dv$$
$$t = \int \frac{m}{F(v)} dv$$

$$F(v) = mv\frac{dv}{dx}$$
$$\int dx = \int \frac{mv}{F(v)} dv$$
$$x = \int \frac{mv}{F(v)} dv$$

E.g. 3 The force acting on a particle of mass 1 kg is given by $F(v) = 9 - v^2$. Given that the particle was initially at rest at the origin, find the value of t when $v = 1$.

Working: Since we need an equation involving t, use $a = \dfrac{dv}{dt}$…

Vertical motion

When an object is moving in a vertical straight line its weight must also be taken into account when using Newton's 2nd law.

E.g. 4 A ball of mass 0.25 kg is projected vertically upwards from ground level with an initial speed of 20 m/s. A resisting force of magnitude $0.05v$ N acts on P during its ascent, where v m/s is the speed and x m is the displacement of the ball at time t s after it starts to move.

(a) Find an expression for $\dfrac{dv}{dt}$ in terms of v.

(b) Find an expression for v as a function of t.

(c) Find an expression for x as a function of t.

(d) Find the greatest height reached by the ball.

Power under a variable force

Power $= Fv$

E.g. 5 A motorcycle with its rider has mass 300 kg. The power of the engine is 5 kW, and the air resistance is given by $0.5v^2$ N when the speed is v m/s. Find how far it travels in increasing its speed from 5 m/s to 15 m/s.

Video (password needed): Force as a function of time

Video (password needed): Force as a function of displacement

Video (password needed): Force as a function of velocity (Example 1)

Video (password needed): Force as a function of velocity (Example 2)

Solutions to Starter and E.g.s

Exercise

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Summary

Choose the correct version of acceleration to fit the model: either $a = \dfrac{dv}{dt}$ ***or*** $a = v\dfrac{dv}{dx}$.

Force as a function of time: $F(t) = m\dfrac{dv}{dt}$ gives $v = \dfrac{1}{m}\int F(t)dt$

Force as a function of displacement: $F(x) = mv\dfrac{dv}{dx}$ gives $\dfrac{1}{2}mv^2 = \int F(x)dx$

Force as a function of velocity: $F(v) = m\dfrac{dv}{dt}$ gives $x = \int \dfrac{mv}{F(v)}dv$

and $F(v) = mv\dfrac{dv}{dx}$ gives $t = \int \dfrac{m}{F(v)}dv$