Work done by a variable force

Starter

1. **(Review of last lesson)** A uniform rectangular lamina $ABCD$ is such that $AB = 0.5$ m and $BC = 0.3$ m. The lamina is placed vertically on a rough inclined plane. Find the maximum angle of inclination, and the least coefficient of friction for which the lamina can rest in equilibrium without toppling or sliding, if the side in contact with the plane is
   (a) BC
   (b) AB.

2. An increasing horizontal force P newtons is applied to a block of mass 4 kg whose height is 40 cm and whose length is 70 cm until the block moves. The coefficient between the block and horizontal plane is 0.7. Find the magnitude of P if the block is:
   (a) on the point of sliding
   (b) on the point of toppling.
   (c) Does the block slide or topple?

(Review of AS FM material)

Work done = force × distance

3. A box of mass 6 kg is pulled at a constant speed of 4 m/s for 20 seconds by an inelastic rope along a horizontal surface. Find the work down if the tension in the rope is 18 N.

4. Find the work down against gravity if a workman of mass 87 kg climbs a vertical ladder of length 7 m.

5. A block of mass 14 kg is pulled 6 m up a plane inclined at 20° to the horizontal. The contact is rough and the magnitude of the frictional force is 30 N. Assuming constant speed, find the work down against:
   (a) friction
   (b) gravity.

Notes

The formula work done = force × distance depends on the force being constant.
What happens if the force is not constant i.e. it is variable?
If an object is moved in a straight line from position x_1 to x_2 by the action of a variable force $f(x)$ that depends on displacement, x, work done is defined as $\int_{x_1}^{x_2} f(x)dx$.

E.g. 1 A car of mass 1020 kg moves from rest at A on a horizontal road. The driving force is constant at 1800 N and resistance to motion is modelled as $\frac{x^2}{8}$ N. The car moves 120 m to B.

(a) Find the work down by the driving force and the work done against resistance as the car travels.

(b) Find the speed of the car at B.

Working:

(a) WD by driving force $= 1800 \times 120 = 216000$ J

WD against resistance = $\int_0^{120} \frac{x^2}{8} dx = 72000$ J

(b) Increase in KE $= 216000 - 72000 = 144000$ J

So $\frac{1}{2} \times 1020 \times v^2 = 144000$

$\therefore v = \frac{40\sqrt{51}}{17} \approx 16.8$ m/s

The speed of the car at B is 16.8 m/s (3 s.f.).

E.g. 2 An object is moving in a horizontal line against a resistive force that is directly proportional to its distance from its starting point, $f(x) = kx$. If the work done against resistance as the object travels from the origin to a point 15 m away is 337.5 kJ,

(a) find the magnitude of k

(b) state the units of k.

Video: Work done by a variable force

Video (password needed): Work done by a variable force

Solutions to Starter and E.g.s

Exercise

p141 6A Qu 1-10, (11-12 red)

Summary

If an object is moved in a straight line from position x_1 to x_2 by the action of a variable force $f(x)$ that depends on displacement, x, work done is defined as $\int_{x_1}^{x_2} f(x)dx$.