

Write yours and your teacher's name at the top of your answer sheets.

U6 Further Mathematics Mock Paper 2 (Mechanics/Statistics)

Answers should be written on file paper.

*Students – please start a new sheet of
paper when you start the statistics
section.*

**February 2022
2021-2022**

Duration: 1 hour 30 minutes

Total number of marks: 75

You are permitted to use a scientific or graphical calculator in this paper.

Students need a formula booklet.

Final answers should be given to a degree of accuracy appropriate to the context.

Mechanics [41]

1.

A particle P of mass 4.5 kg is moving in a straight line on a smooth horizontal surface at a speed of 2.4 ms^{-1} when it strikes a vertical wall directly. It rebounds at a speed of 1.6 ms^{-1} .

- (a) Find the coefficient of restitution between P and the wall. [1]
- (b) Determine the impulse applied to P by the wall, stating its direction. [3]
- (c) Find the loss of kinetic energy of P as a result of the collision. [2]
- (d) State, with a reason, whether the collision is perfectly elastic. [1]

2.

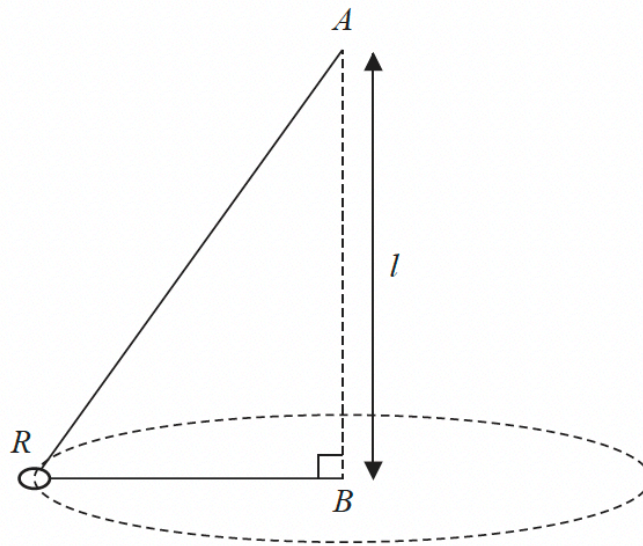


Figure 4

One end of a light inextensible string of length $2l$ is attached to a fixed point A . A small smooth ring R of mass m is threaded on the string and the other end of the string is attached to a fixed point B . The point B is vertically below A , with $AB = l$. The ring is then made to move with constant speed V in a horizontal circle with centre B . The string is taut and BR is horizontal, as shown in Figure 4.

- (a) Show that $BR = \frac{3l}{4}$ [2]

Given that air resistance is negligible,

- (b) find, in terms of m and g , the tension in the string, [4]
- (c) find V in terms of g and l . [4]

3.

Fig. 7.1 shows a uniform lamina in the shape of a sector of a circle of radius r and angle 2θ where θ is in radians. The sector consists of a triangle OAB and a segment bounded by the chord AB .

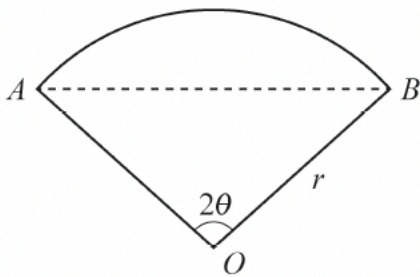


Fig. 7.1

- (a) Explain why the centre of mass of the **segment** lies on the radius through the midpoint of AB . [1]
- (b) Show that the distance of the centre of mass of the **segment** from O is $\frac{2r \sin^3 \theta}{3(\theta - \sin \theta \cos \theta)}$. [4]

A uniform circular lamina of radius 5 units is placed with its centre at the origin, O , of an x - y coordinate system. A component for a machine is made by removing and discarding a segment from the lamina. The radius of the circle from which the segment is formed is 3 units and the centre of this circle is O . The centre of the straight edge of the segment has coordinates $(0, 2)$ and this edge is perpendicular to the y -axis (see Fig. 7.2).

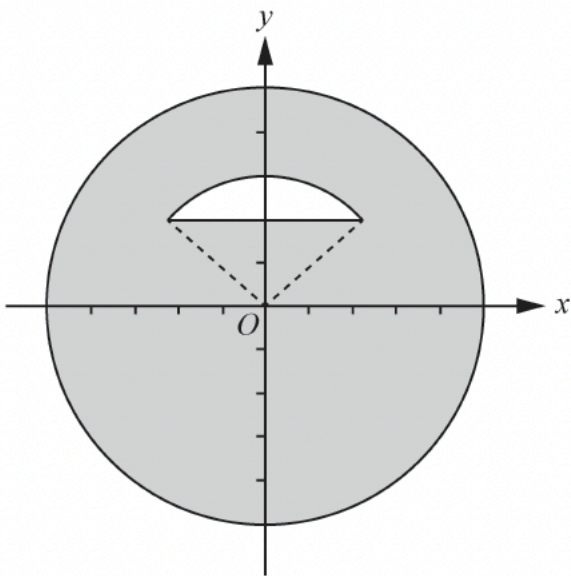


Fig. 7.2

- (c) Find the y -coordinate of the centre of mass of the component, giving your answer correct to 3 significant figures. [5]

4.

Three particles of masses $3m$, $4m$ and $2m$ are placed at the points $(-2, 2)$, $(3, 1)$ and (p, p) respectively.

The value of p is such that the distance of the centre of mass of the three particles from the point $(0, 0)$ is as small as possible.

Find the value of p .

(7)

5.

A particle, P , of mass m kg is projected with speed 5 m s^{-1} down a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{3}{5}$

The total resistance to the motion of P is a force of magnitude $\frac{1}{5} mg$

Use the work-energy principle to find the speed of P at the instant when it has moved a distance 8 m down the plane from the point of projection.

(7)

Statistics [34]

6.

Jo can use either of two different routes, A or B, for her journey to school. She believes that route A has shorter journey times. She measures how long her journey takes for 17 journeys by route A and 12 journeys by route B. She ranks the 29 journeys in increasing order of time taken, and she finds that the sum of the ranks of the journeys by route B is 219.

(a) Test at the 10% significance level whether route A has shorter journey times than route B. [8]

(b) State an assumption about the 29 journeys which is necessary for the conclusion of the test to be valid. [1]

7.

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{114}(4x + 7) & 0 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the median of X is 3.87, correct to three significant figures.

[3 marks]

(b) Find the exact value of $P(X > 2)$

[2 marks]

(c) The continuous random variable Y has probability density function

$$g(y) = \begin{cases} \frac{1}{2}y^2 - \frac{1}{6}y^3 & 1 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(c) (i) Show that $\text{Var}\left(\frac{1}{Y}\right) = \frac{2}{81}$

[4 marks]

(c) (ii) The random variable X has variance $\frac{939}{361}$ and is independent of Y

Find $\text{Var}\left(2X - \frac{3}{Y}\right)$, giving your answer to three significant figures.

[2 marks]

8.

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x \leq 1 \\ \frac{1}{10}x - \frac{1}{10} & 1 < x \leq 6 \\ \frac{1}{90}x^2 + \frac{1}{10} & 6 < x \leq 9 \\ 1 & x > 9 \end{cases}$$

(a) Find the probability density function $f(x)$

[2 marks]

(b) Show that $\text{Var}(X) = \frac{6737}{1200}$

[4 marks]

9.

The distance, X metres, between successive breaks in a water pipe is modelled by an exponential distribution. The mean of X is 25

The distance between two successive breaks is measured. A water pipe is given a 'Red' rating if the distance is less than d metres.

The government has introduced a new law changing d to 2

Before the government introduced the new law, the probability that a water pipe is given a 'Red' rating was 0.05

- (a) Explain whether or not the probability that a water pipe is given a 'Red' rating has increased as a result of the new law. **[4 marks]**
- (b) Find the probability density function of the random variable X . **[2 marks]**
- (c) After investigation, the distances between successive breaks in water pipes are found to have a standard deviation of 5 metres.

Explain whether or not the use of an exponential model in parts (a) and (b) is appropriate.

[2 marks]