U6 FM Mock (Mechanics/Statistics) 21-22 SOLUTIONS [75]

1.

(a)		$1.6 = e \times 2.4 \Rightarrow e = \frac{2}{3}$	B1 [1]	1.1		
(b)		$4.5 \times -1.6 - 4.5 \times 2.4$ $= -18$ so 18 Ns (or kg ms⁻¹)… …in the final direction of motion of *P*	M1 A1 B1 [3]	1.1 1.1 2.2a	Attempt at $mv - mu$ Allow ± 18 could be shown on a diagram	Allow sign confusion e.g. $1.6 - 2.4$ Ignore missing units
(c)		$\frac{1}{2} \times 4.5 \times 2.4^2 - \frac{1}{2} \times 4.5 \times 1.6^2$ 7.2 J	M1 A1 [2]	1.1 1.1	Attempt at $\pm\left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right)$	
(d)		Not perfectly elastic since KE is lost (due to the collision)	B1 [1]	1.2	or $e < 1$ but valid reason must be given.	Must mention KE or collision, Not just e.g. "energy lost"

2.

(a)	$l^2 + r^2 = (2l - r)^2$, using Pythagoras	M1
	$BR = \frac{3l}{4}$ *	A1*
		(2)
(b)	Resolve vertically	M1
	$T\cos\alpha = mg$	A1
	Overall strategy to solve problem: substitute for $\cos\alpha$ and solve for T	M1
	$T = \frac{5mg}{4}$	A1
		(4)
(c)	Equation of motion horizontally	M1
	$T + T\sin\alpha = \frac{mV^2}{r}$	A1
	Overall strategy to solve problem: substitute for T, $\sin\alpha$ and r and solve for V	M1
	$V = \sqrt{\frac{3gl}{2}}$	A1
		(4)

3.

(a)		The central radius is a line of symmetry of the shape.	B1 [1]	2.4	Allow equal area each side	
(b)		Area/mass of sector $\frac{1}{2}r^2\times 2\theta$ and CoM of sector at $\frac{2r\sin\theta}{3\theta}$ from centre used	B1	1.2	Must be used	
		Area/mass of triangle $(-)\frac{1}{2}r^2\sin(2\theta)$ and CoM of triangle at $\frac{2}{3}r\cos\theta$ from centre used	B1	1.2		
		$\dfrac{\frac{1}{2}r^2\times 2\theta\times\frac{2r\sin\theta}{3\theta}-\frac{1}{2}r^2\sin 2\theta\times\frac{2}{3}r\cos\theta}{\frac{1}{2}r^2\times 2\theta-\frac{1}{2}r^2\sin 2\theta}$	M1	3.1b	Using $\bar{x}=\frac{\Sigma m_i x_i}{\Sigma m_i}$ with sector and triangle of –ve mass, oe	
		$=\frac{4r\sin\theta-2r\sin 2\theta\cos\theta}{3(2\theta-\sin 2\theta)}$ $=\frac{2r(\sin\theta-\sin\theta\cos^2\theta)}{3(\theta-\sin\theta\cos\theta)}=\frac{2r\sin\theta(1-\cos^2\theta)}{3(\theta-\sin\theta\cos\theta)}$ $=\frac{2r\sin^3\theta}{3(\theta-\sin\theta\cos\theta)}$	A1 [4]	1.1	AG At least one intermediate step must be seen.	Must see change from double angle
(c)		Sector angle is $2\cos^{-1}\frac{2}{3}$ or $\theta=\cos^{-1}\frac{2}{3}$	B1	2.2a	1.682…	0.841…
		Area/Mass of component: $\pi\times 5^2-\frac{1}{2}\times 3^2(1.682-\sin 1.682)$	M1	1.1	Attempting to find mass of component using 'negative' mass	Allow use of 0.841
		$=75.44$	A1	1.1		
		$\bar{y}=\frac{(-3.097\ldots\times 2.406\ldots)}{75.44\ldots}$	M1	3.1b	Their 3.097, 2.406 and 75.44	If $\theta=2\cos^{-1}\frac{2}{3}$ used this gives 1.095… instead of 2.406…
		$=-0.0988$ (3 sf)	A1 [5]	1.1		

4.

Correct method to find an equation in $\bar{x}$	M1	1.1b
$-3\times 2+4\times 3+2\times p=9\bar{x}\quad (6+2p=9\bar{x})$	A1	1.1b
Correct method to find an equation in $\bar{y}$	M1	1.1b
$3\times 2+4\times 1+2\times p=9\bar{y}\quad 10+2p=9\bar{y}$	A1	1.1b
$(9\bar{x})^2+(9\bar{y})^2=(6+2p)^2+(10+2p)^2$ $(=136+64p+8p^2)$	M1	1.1b
$=8\left[(p+4)^2+17-16\right]$	M1	3.1a
$\Rightarrow p=-4$	A1	2.2a

5.

Work done $= \frac{1}{5}mg \times 8 \quad (15.68m)$	B1
PE Loss $= \quad 8mg\sin\alpha \quad (47.04m)$	B1
KE Gain = Difference of two KE terms	M1
$= \frac{1}{2}mv^2 - \frac{1}{2}m5^2$	A1
Work done against friction = PE Loss – KE Gain	M1
$\frac{1}{5}mg \times 8 = 8mg\sin\alpha - \left(\frac{1}{2}mv^2 - \frac{1}{2}m5^2\right)$	A1
$v = 9.4$ or 9.37 (m s^{-1})	A1
	(7)

5.

(a)		H_0: $m_A = m_B$, H_1: $m_A < m_B$ where m_A and m_B are the median journey times for A and B	**B1**	1.1	*OR:* Median journey times equal, oe. Allow if ms used but not defined	Allow "mean" or "average" only if "population" stated
		$W \sim N(180, 510)$	**B1**	1.1	Both, can be implied, needs $m = 12$	Allow $\sqrt{510}$ or 510^2
		Consider correct tail, either 219 or 141 ($R_m = 219$, $m(m+n+1) - R_m = 141$)	**M1**	1.1	Find *either* $P(\geq 219)$ (218.5) *or* $P(\leq 141)$ (141.5)	Use of 0.9559 is M0 here. For CV method see below
		$p = \Phi\left(\frac{141.5-180}{\sqrt{510}}\right) = 0.0441...$ **BC**	**M1** **A1**	1.1 1.1	Needs *some* evidence. E.g.: 0.0421, 0.0401, 0.470 (no/wrong cc, √): M1	
		$0.0441 < 0.1$	**A1ft**	1.1	Explicit comparison. FT on wrong p-value provided method correct	$0.9559 > 0.9$: A1A1 (M1A1) $0.9559 > 0.1$: A1A0 M0A0
	OR:	CV $180 - z \times \sqrt{510}$	**M1**		Allow √ errors	$180 + 1.282\sqrt{510}$ etc is M0
		141 (141.5) used	**M1**			*unless* 219 (218.5) used, in
		$z = 1.282$ (CV = 151.05, 151.058..)	**A1**		Stated or implied	which case give M2(A1A1)
		$141.5 < 151.05(85)$ or $218.5 > 208.95$	**A1**		CV and cc correct e.g. $141 < 150.55$	E.g. $219 > 209.45$
		Reject H_0.	**M1ft**	1.1	Correct first conclusion	Needs like-with-like, e.g.
		Significant evidence that route B takes longer	**A1ft** **[8]**	2.2b	Contextualised, not too definite	0.9559 with 0.9
		SC Sum of A's ranks = 435 – 219 = 216 used: B1B0 M0M1A0A1 M1A1 max 5/8				
	Exx	α: H_0: Journey times are the same, H_1: journey times for B are higher: B0 β: H_0: No evidence that median journey times are different, etc: B0				
(b)		Must be a random sample (of all journeys) *Or* distributions must be same shape (necessary assumption for Wilcoxon rank-sum test!)	**B1** **[1]**	3.5b	Or equivalent. Allow "(journeys) independent"	*Not* "representative".

6.

(a)	Integrates $\int_0^m \frac{1}{114}(4x+7)\,dx$ and equates with 0.5	1.1a	M1	$\int_0^m \frac{1}{114}(4x+7)\,dx = 0.5$ $\frac{1}{114}(2m^2+7m)=0.5$ $\frac{1}{57}m^2+\frac{7}{114}m-0.5=0$ m = 3.8680512 m = 3.87 (3 sig fig)
	Obtains correct quadratic equation in terms of the median equal to zero PI	1.1a	M1	
	Completes rigorous argument to show that median is 3.87 to 3 significant figures. A more accurate answer must be seen and the other solution from the quadratic equation must be rejected if seen.	2.1	R1	

(b)	Uses an integral of $f(x)$ with one limit of 2 PI	1.1a	M1	P(X > 2) $=\int_2^6 \frac{2}{57}x+\frac{7}{114}\,dx$ $=\frac{46}{57}$
	Obtains the correct exact value of P(X > 2)	1.1b	A1	

(c)(i)	Uses the general formula for E$(f(Y))$ to obtain E$\left(\frac{1}{Y}\right)$	1.1a	M1	E$\left(\frac{1}{Y}\right)=\int_1^3 y^{-1}\left(\frac{1}{2}y^2-\frac{1}{6}y^3\right)dy$ $=\frac{5}{9}$ E$\left(\frac{1}{Y^2}\right)=\int_1^3 y^{-2}\left(\frac{1}{2}y^2-\frac{1}{6}y^3\right)dy$ $=\frac{1}{3}$ Var$\left(\frac{1}{Y}\right)$ = E$\left(\frac{1}{Y^2}\right)$ − $\left(\text{E}\left(\frac{1}{Y}\right)\right)^2$ $=\frac{1}{3}-\left(\frac{5}{9}\right)^2$ $=\frac{2}{81}$
	Uses the general formula for E$(f(Y))$ to obtain E$\left(\frac{1}{Y^2}\right)$	1.1a	M1	
	Uses the formula for the variance to obtain an expression for Var$\left(\frac{1}{Y}\right)$	1.1a	M1	
	Completes a rigorous argument to obtain the given value of Var$\left(\frac{1}{Y}\right)$	2.1	A1	

(c)(ii)	Uses correct general formula for the sum of linear functions of independent variables and to obtain an expression for $\text{Var}\left(AX - \frac{B}{Y}\right)$ Condone sign error	1.1a	M1	$\text{Var}\left(2X - \frac{3}{Y}\right)$ $= 2^2\,\text{Var}(X) + 3^2\,\text{Var}\left(\frac{1}{Y}\right)$ $= 4 \times \frac{939}{361} + 9 \times \frac{2}{81}$ $= 10.6$
	Obtains the correct value of $\text{Var}\left(2X - \frac{3}{Y}\right)$ AWRT 10.6	1.1b	A1	

7.

(a)	Differentiates to find either $\frac{1}{10}$ or $\frac{1}{45}x$ or equivalent	1.1a	M1	$f(x) = \begin{cases} \frac{1}{10} & 1 < x \le 6 \\ \frac{1}{45}x & 6 < x \le 9 \\ 0 & \text{otherwise} \end{cases}$
	Obtains fully defined $f(x)$	1.1b	A1	
(b)	Uses integrals of $xf(x)$ to find $E(X)$	1.1a	M1	$E(X) = \int_1^6 \frac{1}{10}x\,dx + \int_6^9 \frac{1}{45}x^2\,dx$ $= 5.55$ $E(X^2) = \int_1^6 \frac{1}{10}x^2\,dx + \int_6^9 \frac{1}{45}x^3\,dx$ $= \frac{437}{12}$ $\text{Var}(X) = \frac{437}{12} - (5.55)^2$ $= \frac{6737}{1200}$
	Uses integrals of $x^2f(x)$ to find $E(X^2)$	1.1a	M1	
	Obtains correct value of $E(X)$ or the correct value of $E(X^2)$ Accept AWRT 36.4 for $E(X^2)$ PI	1.1b	A1	
	Completes rigorous argument to find the given value of $\text{Var}(X)$ using $\text{Var}(X) = E(X^2) - (E(X))^2$	2.1	R1	

8.

(a)	Uses the mean equalling 25 to find the value of λ	3.3	M1	$\frac{1}{\lambda} = 25$ $\lambda = 0.04$ $P(X \leq 2) = 1 - e^{-0.04 \times 2}$ $= 0.077 > 0.05$ The probability of a 'Red' rating has increased.
	Evaluates an exponential model to find $P(X \leq 2)$ or uses it to find x	3.4	M1	
	Find $P(X \leq 2)$ = AWRT 0.077 or x = AWRT 1.3 metres.	1.1b	A1	
	Concludes probability higher or probability will be increased because 2 metres is higher than 1.3 metres. FT 'their' probability or x value.	3.2a	E1F	
(b)	States the correct probability density function for $x \geq 0$ for their value of λ	1.1b	B1F	$f(x) = \begin{cases} 0.04e^{-0.04x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
	States the complete correct probability density function including $x < 0$ (or otherwise).	1.2	B1F	
(c)	Obtains the standard deviation of the exponential model or the value of λ (0.2) corresponding to a standard deviation of 5 metres.	1.1b	B1	s.d. = $\sqrt{\frac{1}{0.04^2}} = 25$ metres The model is not appropriate as the actual standard deviation of 5 metres is very different from the standard deviation of the model.
	Explains that the model is not appropriate as the actual standard deviation is not close to the standard deviation of the model or λ values are different.	3.5b	E1	