Write yours and your teacher's name at the top of your answer sheets.

U6 Further Mathematics Mock

Paper 2 (Mechanics/Statistics)

Answers should be written on file paper.

Students – please start a new sheet of paper when you start the statistics section.

February 2023
2022-2023

Duration: 1 hour 30 minutes

Total number of marks: 75

You are permitted to use a scientific or graphical calculator in this paper.

Students need a formula booklet.

Final answers should be given to a degree of accuracy appropriate to the context.

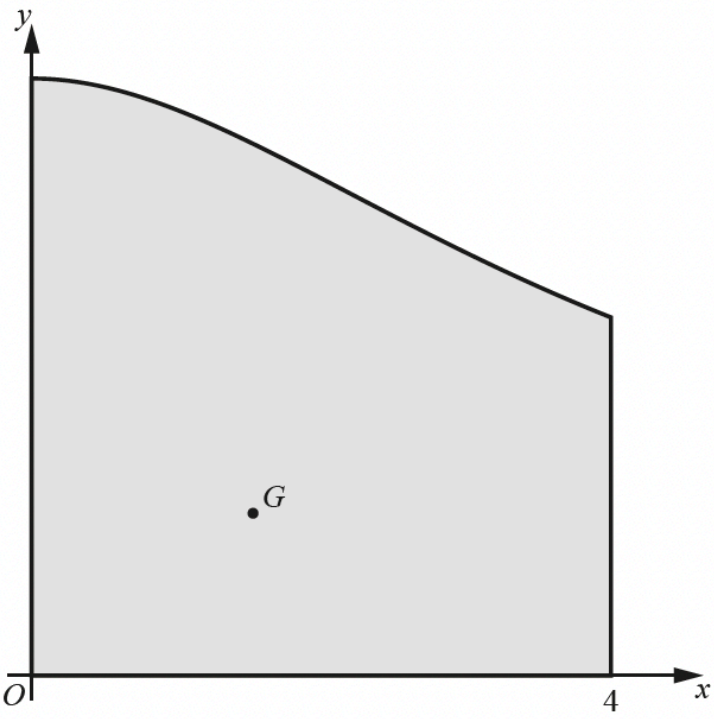
Mechanics [41]

1.

In this question you must show detailed reasoning.

The region bounded by the x-axis, the y-axis, the line $x = 4$ and the curve with equation $y = \dfrac{15}{\sqrt{x^2+9}}$ is occupied by a uniform lamina.

The centre of mass of the lamina is at the point $G(\overline{x}, \overline{y})$ (see diagram).



(a) Show that $\overline{x} = \dfrac{2}{\ln 3}$. **[3]**

(b) Determine the value of $\overline{y}$. Give your answer correct to **3** significant figures. **[3]**

P is the point on the curved edge of the lamina where $x = 3$. The lamina is freely suspended from P and hangs in equilibrium in a vertical plane.

(c) Determine the acute angle that the longest straight edge of the lamina makes with the vertical. **[3]**

2.

Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of P is $3m$ and the mass of Q is $4m$.
Immediately before the collision the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

Show that the speed of Q immediately after the collision is $\dfrac{u}{7}(9e + 2)$

(6)

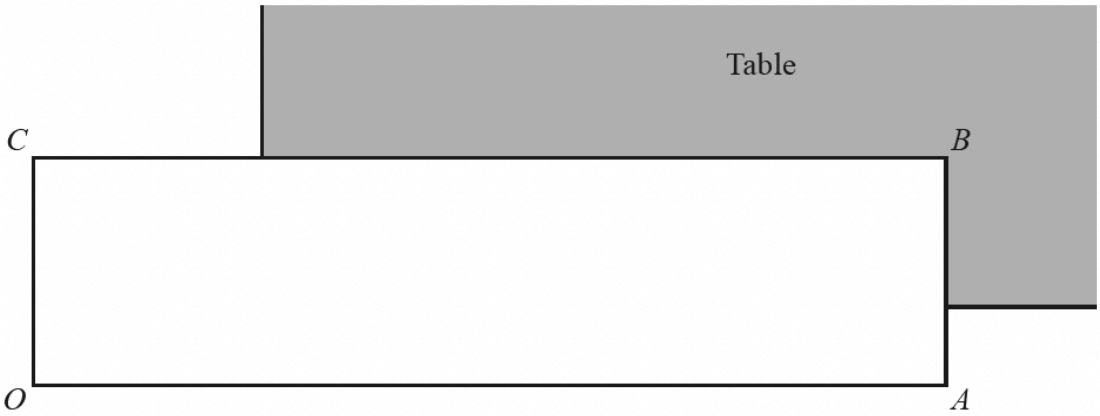
3.

A rectangular lamina of mass M has vertices at the origin O (0, 0), A ($24a$, 0), B ($24a$, $6a$) and C (0, $6a$), where a is a positive constant. A small object P of mass m is attached to the lamina at the point (x, y). The centre of mass of the system consisting of the lamina and P is at the point $(\overline{x}, \overline{y})$. P is modelled as a particle and the lamina is modelled as being uniform.

(a) Show that $\overline{x} = \dfrac{12Ma + mx}{M + m}$. **[1]**

(b) Find a corresponding expression for $\overline{y}$. **[1]**

The lamina, with P no longer attached, is placed on a horizontal rectangular table, with its sides parallel to the edges of the table, and partly overhanging the edges of the table, as shown in the diagram. The corner of the table is at the point ($6a$, $2a$).



When P is placed on the lamina at O, the lamina topples over one of the edges of the table.

(c) Show that $m > \frac{1}{2}M$. **[4]**

The lamina is now put back on the table in the same position as before. P is placed at the point ($12a$, $6a$) on the smooth upper surface of the lamina, and is projected towards O. At a subsequent instant during the motion, P is at the point ($12ak$, $6ak$) where $0 < k < 1$.

(d) Assuming that the lamina has not yet toppled, find, in terms of M and m, the value of k for which the centre of mass of the system lies on the table edge parallel to OC. **[3]**

(e) For the case $m = \frac{3}{2}M$, determine which table edge the lamina topples over. **[4]**

4.

A spring of natural length a has one end attached to a fixed point A. The other end of the spring is attached to a package P of mass m.
The package P is held at rest at the point B, which is vertically below A such that $AB = 3a$.
After being released from rest at B, the package P first comes to instantaneous rest at A.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

(5)

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g.

(6)

Statistics [34] START A NEW SHEET OF PAPER

5.

The critical region for an $r\%$ two-tailed Wilcoxon signed-rank test, based on a large sample of size n, is $\{W_+ \leqslant 113\} \cup \{W_+ \geqslant 415\}$.

(a) Show that $n = 32$. **[3]**

(b) Using a suitable approximation, determine the value of r. **[4]**

6.

The continuous random variable X has cumulative distribution function given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ 1.5x - 0.25x^2 - 1.25 & 1 \leqslant x \leqslant 3 \\ 1 & x > 3 \end{cases}$$

(a) Find the exact value of the median of X

(2)

(b) Find $\mathrm{P}(X < 1.6 \,|\, X > 1.2)$

(3)

The random variable $Y = \frac{1}{X}$

(c) Specify fully the cumulative distribution function of Y

(4)

(d) Hence or otherwise find the mode of Y

(3)

7.

The random variable X has an exponential distribution with probability density function $f(x) = \lambda e^{-\lambda x}$ where $x \geq 0$

(a) Show that the cumulative distribution function, for $x \geq 0$, is given by $F(x) = 1 - e^{-\lambda x}$ **[3 marks]**

(b) Given that $\lambda = 2$, find $P(X > 1)$, giving your answer to three decimal places. **[2 marks]**

8.

In a school opinion poll a random sample of 8 pupils were asked to rate school lunches on a scale of 0 to 20. The results were as follows.

0 1 2 3 4 10 11 13

After a new menu was introduced, the test was repeated with a different random sample of 8 pupils. The results were as follows.

7 8 9 14 15 17 19 20

(a) Carry out an appropriate Wilcoxon test at the 5% significance level to test whether pupils' opinions of school lunches have changed. **[8]**

A statistics student tells the organisers of the opinion poll that it would have been better to have asked the same 8 pupils both times.

(b) Explain why the statistics student's suggestion would produce a better test. **[1]**

(c) State which test should be used if the student's suggestion is followed. **[1]**

(d) You are given that there are 12 870 ways in which 8 different integers can be chosen from the integers 1 to 16 inclusive.

Estimate the number of ways of selecting 8 different digits between 1 and 16 inclusive that have a sum less than or equal to the critical value used in the test in part **(a)**. **[2]**