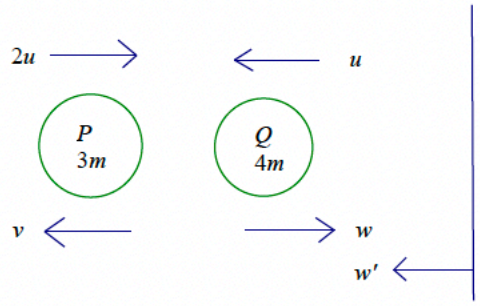
U6 FM Mock (Mechanics/Statistics) 22-23 SOLUTIONS [75]

1.

(a)		DR $\int_0^4 \frac{15}{\sqrt{x^2+9}}dx = 15\left[\ln\left(x+\sqrt{x^2+9}\right)\right]_0^4$ $=15(\ln 9 - \ln 3) = 15\ln 3$	B1	1.1	If this is not seen in **(a)** the mark can be awarded here if integral correctly evaluated in **(b)**.	
		$15\int_0^4 \frac{1}{2}\times 2x(x^2+9)^{-\frac{1}{2}}dx$ $= 15\left[(x^2+9)^{\frac{1}{2}}\right]_0^4$	M1	1.1	Integrating. Ignore limits here.	Must be in the form $k\left[(x^2+9)^{\frac{1}{2}}\right]$
		$=15\left[(4^2+9)^{\frac{1}{2}} - (0^2+9)^{\frac{1}{2}}\right]$ $=15[5-3]=30$ $\Rightarrow \bar{x} = \frac{30}{15\ln 3} = \frac{2}{\ln 3}$	A1 [3]	1.1	AG. Some intermediate working must be seen.	Award for fully complete proof only
(b)		DR $\frac{1}{2}\int_0^4 \left(\frac{15}{\sqrt{x^2+9}}\right)^2 dx = \frac{225}{2}\int_0^4 \frac{1}{x^2+9}dx$	M1	1.1	Correct integral and correct limits	
		$=\frac{225}{2}\times\frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^4$	M1	1.1	Integrating into the form $k\left[\tan^{-1}\frac{x}{3}\right]$	May have used substitution
		$=\frac{75}{2}\tan^{-1}\frac{4}{3}$ $\therefore \bar{y} = \frac{\frac{75}{2}\tan^{-1}\frac{4}{3}}{15\ln 3} = \text{awrt}2.11$	A1 [3]	1.1		
(c)		DR $x=3 \Rightarrow y = \frac{15}{\sqrt{18}}$ oe	B1	3.4	Finding the y coord of P. $=\frac{5}{\sqrt{2}}=\frac{5}{2}\sqrt{2}$ or awrt 3.54	
		$\tan\theta = \frac{3-\frac{2}{\ln 3}}{"\frac{15}{\sqrt{18}}" - "2.1101..."} = \frac{1.17952...}{1.42538...}$ $= 0.8275...$	M1	2.1	$\tan\theta = \frac{\Delta x}{\Delta y}$ or $\frac{\Delta y}{\Delta x}$	
		$\theta = \text{awrt}39.6°$	A1 [3]	2.2a	0.691rads	

2.

	
Using CLM:	M1
$6mu - 4mu = -3mv + 4mw \quad (2u = -3v + 4w)$	A1
Use of impact law	M1
$w + v = e \times 3u$	A1
Complete method to find w	M1
$\begin{cases} 3w + 3v = 9eu \\ -3v + 4w = 2u \end{cases} \Rightarrow 7w = 9eu + 2u, \quad w = \frac{u}{7}(9e+2)$ *	A1*

3.

(a)		$\bar{x} = \frac{12a \times M + x \times m}{M+m} = \frac{12Ma + mx}{M+m}$	B1 [1]	1.1	AG. www	
(b)		$\bar{y} = \frac{3a \times M + y \times m}{M+m} = \frac{3Ma + my}{M+m}$	B1 [1]	1.1		
(c)		If P is at O, $\bar{x} = \frac{12Ma}{M+m}$ and $\bar{y} = \frac{3Ma}{M+m}$	B1ft	3.3	FT their expression for $\bar{y}$	Alternative: B1 for correct expressions for $\bar{x}, \bar{y}$ M1: forming 2 inequalities with $2a$ and $6a$ (must be right way around) M1: simplifying or manipulating both inequalities so that they can be combined or compared A1: fully correct and conclusion www
		$\bar{y} < 2a \Rightarrow 3M < 2M + 2m \Rightarrow m > \frac{1}{2}M$	M1	3.4		
		$\bar{x} < 6a \Rightarrow 12M < 6M + 6m \Rightarrow m > M$	M1	3.4		
		Conclusion: $m > \frac{1}{2}M$	A1 [4]	2.4	AG.	
(d)		$\bar{x} = \frac{12Ma + m \times 12ak}{M+m}$ used	B1	3.3		
		$\frac{12Ma + m \times 12ak}{M+m} = 6a$	M1	3.4	Their $\bar{x}$ equated to $6a$	Ignore working with $\bar{y}$
		$k = \frac{m-M}{2m}$ oe	A1 [3]	1.1	$k = \frac{1}{2}\left(1 - \frac{M}{m}\right)$	Ignore working with $\bar{y}$ unless this affects final answer
(e)		$m = \frac{3}{2}M \Rightarrow k_{OC} = \frac{1}{6}$	B1	3.3	$k_{OC} = \frac{3}{18} = 0.1\dot{6}$	
		$\bar{y} = \frac{3Ma + \frac{3}{2}M \times 6ak}{M + \frac{3}{2}M}$	M1	3.4	Substituting $y = 6ak$ and $m = \frac{3}{2}M$ into their $\bar{y}$	
		$\bar{y} = 2a \Rightarrow \frac{6a + 18ak}{5} = 2a \Rightarrow k_{OA} = \frac{2}{9}$	A1	3.4	$k_{OA} = \frac{4}{18} = 0.\dot{2}$	
		(k changes from 1 to 0 and $k_{OA} > k_{OC}$ so) lamina topples over edge OA.	A1 [4]	2.2a	www	

4.

(a)	EPE at $A = \frac{\lambda a^2}{2a}$ or EPE at $B = \frac{\lambda(2a)^2}{2a}$	M1	2.1
	Form work-energy equation:	M1	3.3
	$\frac{\lambda a^2}{2a} + mg \times 3a = \frac{\lambda(2a)^2}{2a}$ $\left(\frac{\lambda a}{2} + 3mga = 2\lambda a\right)$	A1 A1	1.1b 1.1b
	$3mg = \frac{3\lambda}{2} \Rightarrow \lambda = 2mg$ *	A1*	2.2a
		(5)	
(b)	Extension at equilibrium:	M1	2.1
	$\frac{2mgx}{a} = mg \Rightarrow x = \frac{a}{2}$ *	A1*	1.1b
	Alternative for the first M1A1:		
	Use the work-energy equation to obtain $\frac{\mathrm{d}V^2}{\mathrm{d}x}$ and set the derivative equal to zero	M1	
	$\frac{1}{a} \times 2x - 1 = 0 \Rightarrow x = \frac{a}{2}$	A1	
	Use work-energy equation to find max speed:	M1	3.4
	$\frac{2mgx^2}{2a} + mg \times (2a - x) + \frac{1}{2}mV^2 = \frac{2mg(2a)^2}{2a}$ $\left(\frac{ag}{4} + \frac{3ag}{2} + \frac{1}{2}V^2 = 4ag\right)$	A1 A1	1.1b 1.1b
	$V = 3\sqrt{\frac{ag}{2}}$	A1	2.2a
		(6)	

5.

(a)		$\mu = 264$ $\frac{1}{4}n(n+1) = 264$ $\Rightarrow n = 32$ or -33, but $n > 0$ so 32 only **AG**	**B1** **M1** **A1** **[3]**	3.1a 1.1 2.2a	Allow even if no CC used Use formula for mean Solve $n^2 + n - 1056 = 0$ to obtain 32	Or: $113 + 415 = \frac{1}{2}n(n+1)$: M2 Can be implied by both 32 and -33 Just verification: SC B1
(b)		Variance $= \frac{1}{24}n(n+1)(2n+1) = 2860$ $\Phi(113.5 - 264)/\sqrt{2860}$ $(= 0.002445)$ $\Rightarrow r = 2\times 0.002445 \times 100\%$ $= 0.489$ (%)	**B1** **M1*** **depM1** **A1** **[4]**	3.3 3.4 3.1b 2.2a	Variance 2860 stated or implied Standardise, their parameters (or use 414.5) Double their p-value ($p < 0.5$), oe 0.49 or better (0.48 is probably from no cc)	No or wrong cc: (0.503% or 0.518%): B1M1M1A0 Allow 0.50% *only* if correct CC seen

6.

(a)	$1.5m - 0.25m^2 - 1.25 = 0.5 \quad (\rightarrow 0.25m^2 - 1.5m + 1.75 = 0)$	M1	1.1b
	$m = 3 - \sqrt{2} \qquad$ (reject $m = 3 + \sqrt{2}$)	A1	2.2a
		(2)	
(b)	$\mathrm{P}(X < 1.6 \mid X > 1.2) = \dfrac{\mathrm{P}(1.2 < X < 1.6)}{\mathrm{P}(X > 1.2)}$	M1	3.1a
	$\dfrac{\mathrm{F}(1.6) - \mathrm{F}(1.2)}{1 - \mathrm{F}(1.2)}$	M1	1.1b
	$= \dfrac{32}{81}$	A1	1.1b
		(3)	
(c)	$\mathrm{P}(Y \leqslant y) = \mathrm{P}\left(\dfrac{1}{X} \leqslant y\right)$		
	$= \mathrm{P}\left(X \geqslant \frac{1}{y}\right) = 1 - \mathrm{F}(\frac{1}{y})$	M1	3.1a
	$= 1 - (\frac{1.5}{y} - 0.25(\frac{1}{y})^2 - 1.25)$	M1	1.1b
	$\mathrm{F}(y) = \begin{cases} 0 & y < \frac{1}{3} \\ 2.25 - \frac{1.5}{y} + 0.25(\frac{1}{y})^2 & \frac{1}{3} \leq y \leq 1 \\ 1 & y > 1 \end{cases}$	A1 A1	2.1 1.1b
		(4)	
(d)	$\mathrm{f}(y) = \dfrac{\mathrm{d}}{\mathrm{d}y}(\mathrm{F}(y)) = 1.5y^{-2} - 0.5y^{-3} \rightarrow \mathrm{f}'(y) = -3y^{-3} + 1.5y^{-4}$	M1	3.1a
	$\mathrm{f}'(y) = -3y^{-3} + 1.5y^{-4} = 0$	depM1	1.1b
	$1.5y^{-4} = 3y^{-3} \rightarrow \frac{1.5}{y^4} = \frac{3}{y^3}$		
	[Mode of Y =] 0.5 (since $\mathrm{f}''(0.5) = -48 < 0$)	A1	1.1b
		(3)	

7.

(a)	Uses $\int \lambda e^{-\lambda t}\, dt$ Condone missing dt or using x for t	1.1a	M1	$F(x) = \int_0^x \lambda e^{-\lambda t}\, dt$ $= \left[-e^{-\lambda t}\right]_0^x$ $= 1 - e^{-\lambda x}$
	Obtains correct integrated function May be unsimplified	1.1b	A1	
	Completes reasoned argument by substituting in limits and subtracting correct way round to show that $F(x) = 1 - e^{-\lambda x}$ or by solving $F(0) = 0$ to find the constant of integration Condone missing dt or using x for t in their solution but no other errors must be seen	2.1	R1	
(b)	Obtains $F(1)$ = **AWRT** 0.865 or selects correct integral **PI** by correct final answer	1.1a	M1	$P(X > 1) = 1 - F(1)$ $= e^{-2\times 1}$ $= 0.135$
	Obtains correct value of $P(X > 1)$ **AWRT** 0.135	1.1b	A1	

8.

(a)		H_0: Two samples are from identical populations	**B2**	1.1	If no reference to "populations", maximum B1
		H_1: Two samples are from populations with different median ratings.		1.1	Allow H_0: "identical population medians", H_1: "not identical populations" or "not identical pop medians"
		$R_m = 1 + 2 + 3 + 4 + 5 + 9 + 10 + 11 \ (= 45)$	**M1**	1.1	"Pupils' opinions have not changed", etc: B2
		$W = 45$	**A1**	1.1	If omitted, can still get all other marks
		$8(8 + 8 + 1) - R_m = 91$	**B1**	2.1	
		$W_{crit} = 49$	**B1**	1.1	
		Reject H_0. Significant evidence that there is a difference in	**M1ft**	1.1	FT on TS (< 68) or CV
		median ratings/opinions have changed	**A1ft**	2.2b	FT on TS only. Allow "increased"
			[8]		SC: Sign or paired-sample test, max B2 (hypotheses)
(b)		Eliminate the difference between individual pupils' opinions	**B1**	3.5b	"Minimises the difference in tastes" B1 (BOD)
			[1]		Scores arbitrary: B1 (etc). Not "more powerful test".
(c)		A paired-sample signed-rank test would have been used	**B1**	3.5c	Must mention "paired sample" oe – not just "Wilcoxon
			[1]		signed rank"
(d)		0.025×12870	**M1**	3.1a	$0.05 \times 12870 = 643.5$ M1
		$= 322$	**A1**	3.2a	321 or 322 or 643 (from 1-tail), must be integer
			[2]		