

*Write yours and your teacher's name at the top of your answer sheets.*

# **U6 Further Mathematics Mock Paper 2 (Mechanics/Statistics)**

*Answers should be written on file paper.*

*Students – please use a new sheet of paper  
when you start the statistics section.*

**February 2024  
2023-2024**

**Duration: 1 hour 30 minutes**

**Total number of marks: 71**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Students need a formula booklet.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

## Mechanics

1.

Three particles of masses  $4m$ ,  $2m$  and  $km$  are placed at the points with coordinates  $(-3, -1)$ ,  $(6, 1)$  and  $(-1, 5)$  respectively.

Given that the centre of mass of the three particles is at the point with coordinates  $(\bar{x}, \bar{y})$

(a) show that  $\bar{x} = \frac{-k}{k+6}$  (3)

(b) find  $\bar{y}$  in terms of  $k$ . (2)

Given that the centre of mass of the three particles lies on the line with equation  $y = 2x + 3$

(c) find the value of  $k$ . (2)

A fourth particle is placed at the point with coordinates  $(\lambda, 4)$ .

Given that the centre of mass of the four particles also lies on the line with equation  $y = 2x + 3$

(d) find the value of  $\lambda$ . (2)

2.

Two identical spheres,  $A$  and  $B$ , each of mass  $4\text{ kg}$ , are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Before they collide, the speeds of  $A$  and  $B$  are  $5\text{ ms}^{-1}$  and  $3\text{ ms}^{-1}$  respectively. Immediately after they collide, the speed of  $A$  is  $2\text{ ms}^{-1}$  and its direction of motion has been reversed.

(a) (i) Determine the velocity of  $B$  immediately after  $A$  and  $B$  collide. [3]

(ii) Show that the coefficient of restitution between  $A$  and  $B$  is  $\frac{3}{4}$ . [2]

(iii) Calculate the total loss of kinetic energy due to this collision. [2]

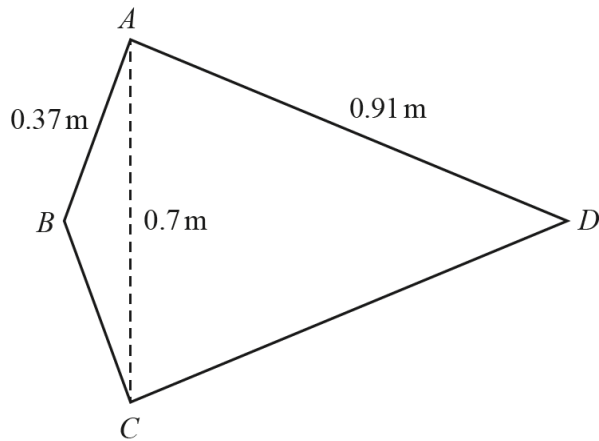
Sphere  $B$  goes on to strike a fixed wall directly. As a result of this collision  $B$  moves along the same straight line with a speed of  $4\text{ ms}^{-1}$ .

(b) Find the coefficient of restitution between  $B$  and the wall, stating whether the collision between  $B$  and the wall is perfectly elastic. [2]

(c) Determine the magnitude of the impulse that  $B$  exerts on  $A$  the next time that they collide. [5]

3.

$ABCD$  is a uniform lamina in the shape of a kite with  $BA = BC = 0.37$  m,  $DA = DC = 0.91$  m and  $AC = 0.7$  m (see diagram). The centre of mass of  $ABCD$  is  $G$ .

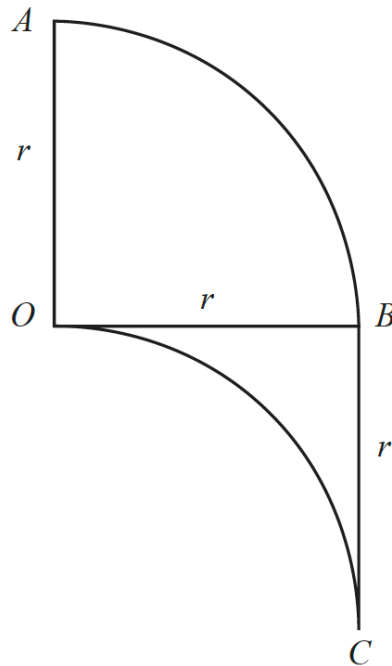


(a) Explain why  $G$  lies on  $BD$ . [1]

(b) Show that the distance of  $G$  from  $B$  is 0.36 m. [4]

4.

[In this question you may quote, without proof, the formula for the distance of the centre of mass of a uniform circular arc from its centre.]



**Figure 1**

Five pieces of a uniform wire are joined together to form the rigid **framework**  $OABCO$  shown in Figure 1, where

- $OA$ ,  $OB$  and  $BC$  are straight, with  $OA = OB = BC = r$
- arc  $AB$  is one quarter of a circle with centre  $O$  and radius  $r$
- arc  $OC$  is one quarter of a circle of radius  $r$
- all five pieces of wire lie in the same plane

(a) Show that the centre of mass of arc  $AB$  is a distance  $\frac{2r}{\pi}$  from  $OA$ .

**(2)**

Given that the distance of the centre of mass of the framework from  $OA$  is  $d$ ,

(b) show that  $d = \frac{7r}{2(3 + \pi)}$

**(4)**

## Statistics

5.

The random variable  $X$  has a continuous uniform distribution over the interval  $[-3, k]$

$$\text{Given that } P(-4 < X < 2) = \frac{1}{3}$$

(a) find the value of  $k$

**(3)**

A computer generates a random number,  $Y$ , where

- $Y$  has a continuous uniform distribution over the interval  $[a, b]$
- $E(Y) = 6$
- $\text{Var}(Y) = 192$

The computer generates 5 random numbers.

(b) Calculate the probability that at least 2 of the 5 numbers generated are greater than 7.5

**(6)**

6.

An historian has reason to believe that the average age at which men got married in the seventeenth century was higher in urban areas compared to rural areas. The historian collected data from a random sample of 8 men in an urban area and a random sample of 6 men in a rural area, all of whom were married in the seventeenth century. The results were as follows, given in the form years/months.

Urban:	18/3	18/5	19/9	20/7	25/6	34/6	41/8	46/3
Rural:	18/0	18/1	18/4	19/11	22/2	28/11		

- (a) Use an appropriate non-parametric method to test at the 5% significance level whether the average age at marriage of men is higher in urban areas than in rural areas. **[7]**
- (b) When checking the data, the historian found that the age of one of the men, Mr X, which had been recorded as 28/11, had been wrongly recorded. When corrected, the result of the test in part (a) was unchanged.

Determine the youngest age that Mr X could have been, given that it was not the same, in years and months, as that of any of the other men in the sample. **[3]**

7.

The continuous random variable  $X$  has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ k(x - ax^2) & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

The values of  $a$  and  $k$  are positive constants such that  $P(X < 2) = \frac{2}{3}$

(a) Find the exact value of the median of  $X$

(6)

(b) Find the probability density function of  $X$

(2)

(c) Hence, deduce the value of the mode of  $X$ , giving a reason for your answer.

(2)

8.

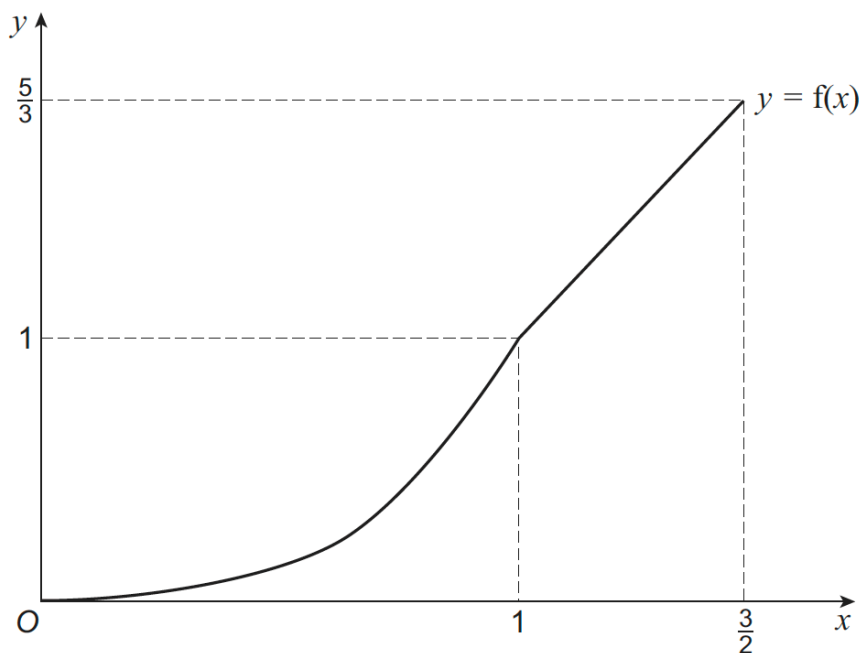
The continuous random variable  $X$  has probability density function  $f(x)$

It is given that  $f(x) = x^2$  for  $0 \leq x \leq 1$

It is also given that  $f(x)$  is a linear function for  $1 < x \leq \frac{3}{2}$

For all other values of  $x$ ,  $f(x) = 0$

A sketch of the graph of  $y = f(x)$  is shown below.



Show that  $\text{Var}(X) = 0.0864$  correct to three significant figures.

[8 marks]