U6 FM Mock (Mechanics/Statistics) 23-24 SOLUTIONS [34 + 37 = 71]

1.

1a	Moments about y axis:	M1	2.1
	$km\times -1+4m\times -3+2m\times 6=(k+6)m\times\bar{x}$	A1	1.1b
	$\Rightarrow \bar{x}=\dfrac{-k}{k+6}$ *	A1*	1.1b
		(3)	
1b	Moments about x axis: $\left(km\times 5+4m\times -1+2m\times 1=(k+6)m\times\bar{y}\right)$	M1	3.1a
	$\Rightarrow \bar{y}=\dfrac{5k-2}{k+6}$	A1	1.1b
		(2)	
1c	$\bar{y}=2\bar{x}+3\Rightarrow \dfrac{5k-2}{k+6}=\dfrac{-2k}{k+6}+3$ $\quad\left(5k-2=-2k+(3k+18)\right)$	M1	1.1b
	$\Rightarrow k=5$	A1	1.1b
		(2)	
1d	$4=2\lambda+3\Rightarrow$	M1	3.4
	$\lambda=\dfrac{1}{2}$	A1	1.1b
		(2)	
			(9 Marks)

2.

(a)	**(i)**	$5m + (-3)m = (-2)m + mv_B$	**M1**	3.4	Conservation of momentum	u_A must be $> u_B$ oe
		$v_B = 4$	**A1**	1.1		
		in the direction of motion of A before the collision oe	**A1**	1.1	Must be clearly stated or shown, e.g. consistent with arrow on diagram	Direction of B is reversed
			[3]			
	(ii)	$e = (4 - (-2)) / (5 - (-3))$	**M1**	3.4	Attempt at restitution - condone sign error as long as consistent	$0 \le (\pm)e \le 1$ Must have sufficient detail, e.g. 6/8 on its own is M0.
		$= 6/8 = ¾$	**A1**	1.1	AG	
			[2]			
	(iii)	Initial KE = ½×4×5^2 + ½×4×(–3)^2 = 68 (J) Final KE = ½×4×(–2)^2 + ½×4×4^2 = 40 (J)	**M1**	3.4	Attempt to calculate total initial or final KE Both values must be positive	Or KE loss for A = ½×4×5^2 – ½×4×(–2)^2 = 42 J or KE gain for B = ½×4×4^2 – ½×4×3^2 = 14 J
		so loss is 68 – 40 = 28 (J)	**A1**	1.1		42 J – 14 J = 28 J
			[2]			
(b)		$e = 4 / 4 = 1$	**B1FT**	1.1	FT their v_B provided that $0 < e \le 1$ (using their values in (a))	Allow e = 1 without working, provided (a)(i) is correct
		The collision is perfectly elastic.	**B1FT**	1.2	FT their e provided that $0 < e \le 1$ (using their values in (a))	Do not accept phases such as "completely elastic"
			[2]			
(c)		$(-2)m + (-4)m = mV_A + mV_B$	**M1***	3.1b	Conservation of momentum with consistent signs. $2V_A + 2V_B = -12$	(if "positive" direction reversed: $2m + 4m = mV_A + mV_B$) M0 if approach speed < 0
		$¾ = (V_B - V_A) / ((-2) - (-4))$	**M1***	3.1b	Restitution with consistent signs $2V_B - 2V_A = 3$	Allow use of e (e.g. $V_B - V_A = \pm 2e$)
		$2V_A + 2V_B = -12$ $2V_B - 2V_A = 3$ $V_B = \ldots$ or $V_A = \ldots$	**M1dep**	1.1	Attempt to solve both their equations simultaneously and reach a solution for V_A or V_B	Allow use of e (e.g. $V_A = \pm(3 + e)$ or $V_B = \pm(3 - e)$)
		$V_B = -2.25$ or $V_A = -3.75$	**A1**	1.1		A numerical value is required here (may be implied by a correct final answer).
		Impulse on A = change in A's momentum $= 4(-3.75 - (-2)) = -7 \Rightarrow 7$(Ns)	**A1**	1.1	Ignore wrong units	ISW e.g. any statements regarding direction of travel
			[5]			

3.

(a)		(BD is a line of) symmetry (of the kite)	**B1**	2.1		
			[1]			
(b)		$h_{ABC} = \sqrt{0.37^2 - 0.35^2} = 0.12$ so area = 0.042	**B1**	3.1b		ALT (considering the CoM of triangle ABD) with X as the point where the diagonals meet: $h_{ABX} = 0.12$ so area = 0.021
		$h_{ADC} = \sqrt{0.91^2 - 0.35^2} = 0.84$ so area = 0.294	**B1**	1.1		$h_{AXC} = 0.84$ so area = 0.147
		Measuring from B: $(0.042 + 0.294)\bar{x}$ $= 0.042 \times 0.08 + 0.294 \times (0.12 + 0.28)$	**M1**	1.1	Attempt to balance moments about any point with at least two non-zero terms each comprising the product of a force or mass with an appropriate distance. May be one error in the distance. Measuring from AC $(0.042 + 0.294)\bar{x}$ $= -0.04 \times 0.042$ $+ 0.28 \times 0.294$ Gives $\bar{x} = \frac{0.08064}{0.336} = 0.24$ Final step: add 0.12 to measure from B	Measuring from B: $(0.021 + 0.147)\bar{x}$ $= 0.021 \times \frac{2}{3} \times 0.12$ $+ 0.147$ $\times \left(0.12 + \frac{1}{3} \times 0.84\right)$
		$\bar{x} = \frac{0.12096}{0.336} = 0.36$	**A1**	1.1	AG. Some intermediate working must be seen.	$\bar{x} = \frac{0.06048}{0.168} = 0.36$ AG
			[4]			
(b)		**Alternative method**				
		E.g. Taking B as (0,0) $h_{ABC} = \sqrt{0.37^2 - 0.35^2} = 0.12$ so A (0.12, 0.35)	**B1**	1.1	Need to see clear coordinates, may be on diagram. Award if average of trianlge coords method used	
		$h_{ADC} = \sqrt{0.91^2 - 0.35^2} = 0.84$ so D (0.96,0)	**B1**	1.1		
		From B, $\bar{x} = \frac{1}{3}(0 + 0.12 + 0.96)$	**M1**	1.1		
		$= \frac{1.08}{3} = 0.36$ as triangle BAD and triangle BCD are symmetrical	**A1**	1.1	AG. Intermediate working and mention of symmetry along BD must be seen.	
			[4]			

4.

(a)	$\alpha = \frac{\pi}{4} \Rightarrow \frac{r\sin\alpha}{\alpha} = r \times \frac{1}{\sqrt{2}} \times \frac{4}{\pi}\left(= \frac{2\sqrt{2}r}{\pi}\right)$	B1
	Distance from $OA = \frac{2\sqrt{2}r}{\pi} \times \cos\frac{\pi}{4} = \frac{2\sqrt{2}r}{\pi \times \sqrt{2}} = \frac{2r}{\pi}$ *	B1*
		(2)
(b)	Moments about OA:	M1
	$r \times \frac{r}{2} + r \times r + \frac{2r}{\pi} \times \frac{\pi r}{2} + \frac{2r}{\pi} \times \frac{\pi r}{2} = (3r + \pi r)d$	A1 A1
	$\left(\frac{7r^2}{2} = (3r + \pi r)d\right) \Rightarrow d = \frac{7r}{2(3+\pi)}$ *	A1*
		(4)

Statistics [37]

5.

(a)	$\mathrm{P}(-4<X<2)=\mathrm{P}(-3<X<2)$	M1
	$\frac{2+3}{k+3}=\frac{1}{3}$ <u>or</u> $k=2+2\times(2--3)$	M1
	$k=12$	A1
		(3)
b)	$\frac{a+b}{2}=6$ **and** $\frac{1}{12}(b-a)^2=192$	M1
	$\frac{1}{12}(a-(12-a))^2=192$ <u>or</u> $\frac{1}{12}((12-b)-b)^2=192$ (oe)	M1
	$a=-18$ <u>or</u> $b=30$	A1
	$\mathrm{P}(Y>7.5)=\frac{"30"-7.5}{"30"-("-18")}$ $\left[=\frac{15}{32}\text{ or } 0.46875 \text{ (accept 0.469 or better)}\right]$	M1
	$R\sim \mathrm{B}(5,"0.46875")$	M1
	$\mathrm{P}(R\geqslant 2)=0.7710...$	A1
		(6)

6.

5	**(a)**	H_0: distributions of ages for urban and rural areas are identical; H_1: distributions differ as to median *Or* H_0: $m_d = 0$, H_1: $m_d > 0$, where m_d is the median of population differences of age of marriage for men in urban and rural areas *or* H_0: population median ages in urban & rural areas are equal, H_1: median age higher in urban areas	**B1**	1.1	If "distributions identical", allow any type of average. If not "distributions identical" then must use median. Allow use of m_u and m_r, provided they are defined as median, or entirely verbal (see below) but need some context
		Rankings of rural ages are 1, 2, 4, 7, 9, 11	**M1**	3.1b	Find rankings of rural ages within whole sample, can be implied by 34
		$R_m = 34$ and $m(m+n+1) - R_m = 56$	**B1**	1.1	Consider $m(m+n+1) - R_m$ (if omitted, can get all other marks)
		$W = 34$	**A1**	1.1	Correct W, allow $p = 0.0876$ from N(45, 60) with cc
		CV 31	**B1**	1.1	Correct CV used, allow 31 or 31.76 from N(45, 60)
		$34 > 31$ so do not reject H_0, *or* $0.0876 > 0.05$	**M1ft**	1.1	Comparison and correct first conclusion, needs correct method for R_m. FT on their 34, but not 56, and on 29 instead of 31 but no other CV except FT on wrong m or n (e.g. $m = 5$, CV 23 which gets B1M1B0A1B0M1A1)
		Insufficient evidence that average ages are higher	**A1ft** **[7]**	2.2b	Contextualised, not over-assertive, not "significant evidence that average ages are the same". Same ft. Allow "different".
If from valid method their $W \le 31$, so that conclusion changes, FT for **(a)** ("significant evidence that average ages are different") and *consult PE* if **(b)** is problematic					
	(b)	$1 + 2 + 4 + 7 + 8 + 10 = 32$	**M1**	3.1b	Attempt to change total to 32 (or 30 if CV 29 used), e.g. 8 seen
			A1	1.1	Find age that makes rank sum of 32 (or 31 if 29 used – can't get 30??) (M1A1 can be implied by answer of 20/0, allow 19/12)
	OR	T&I: Test at least two new ages and find R_m for each 19/10 or 20/0 or 19/12 stated	**M1** **A1**		
		11th becomes 7th so least age of Mr X is 19/10	**A1** **[3]**	3.2a	(old 11th becomes new 7th, old 7th becomes new 8th) *Condone* absence of working.

A	H_0: median in urban & rural areas are equal, H_1: median higher in urban areas *(no context – needs "ages" as a minimum)*	B0
B	H_0: average ages in urban & rural areas are equal, H_1: average age higher in urban areas *(minimum context, but "average" not median")*	B0
C	H_0: median ages in urban & rural areas are equal, H_1: median age higher in urban areas *(condone omission of "population")*	B1
D	H_0: median of differences in ages is 0, H_1: greater than 0 *(BOD for their meaning "urban minus rural" rather than "rural minus urban")*	B1
E	H_0: $m_u = m_r$, H_1: $m_u > m_r$ where m_u and m_r are the median ages of marriage for men in urban and rural districts respectively	B1

7.

(a)	$F(2)=\frac{2}{3}$ and $F(4)=1$		
	$k(2-4a)=\frac{2}{3}$ $\quad k(4-16a)=1$	M1	2.1
	$\frac{2}{3(2-4a)}=\frac{1}{(4-16a)}\to a=\frac{1}{10},\quad k=\frac{5}{12}$	M1A1	1.1b 1.1b
	$\frac{5}{12}\left(m-\frac{1}{10}m^2\right)=0.5\to m^2-10m+12=0$	M1M1	3.1a 1.1b
	$m=5-\sqrt{13}$ $\quad (m=5+\sqrt{13}$ reject)	A1	1.1b
		(6)	
(b)	$f(x)=\frac{d}{dx}(F(x))$	M1	1.1b
	$f(x)=\begin{cases}"\frac{5}{12}"(1-"0.2"x) & 0 \leqslant x \leqslant 4\\ 0 & \text{otherwise}\end{cases}$	A1ft	1.1b
		(2)	
(c)	Mode is $X=0$…	B1	2.2a
	…since f(x) is linear with negative gradient. or …f(x) is a decreasing function	dB1	2.4
		(2)	
			(10 marks)

8.

Uses one of the points shown on the graph and gradient $\frac{4}{3}$ to find an equation of the straight-line section of the probability density function **PI**	3.1a	M1	$m = \frac{\frac{5}{3}-1}{\frac{3}{2}-1} = \frac{4}{3}$ $f(x)-1 = \frac{4}{3}(x-1)$ for $1 < x \le \frac{3}{2}$
Obtains a correct expression $\frac{4}{3}x - \frac{1}{3}$ for the straight-line section of the probability density function	1.1b	A1	$f(x) = \frac{4}{3}x - \frac{1}{3}$ for $1 < x \le \frac{3}{2}$ $E(X) = \int_0^1 x^3 dx + \int_1^{\frac{3}{2}} x\left(\frac{4}{3}x - \frac{1}{3}\right) dx$
Uses the formula for $E(g(X))$ to obtain a correct expression using their equation of the straight-line section for $E(X)$ or $E(X^2)$ **PI** Condone missing brackets	1.1a	M1	$= \frac{1}{4} + \frac{61}{72}$ $= \frac{79}{72}$ $E(X^2) = \int_0^1 x^4 dx + \int_1^{\frac{3}{2}} x^2\left(\frac{4}{3}x - \frac{1}{3}\right) dx$
Obtain correct expressions for $E(X)$ and $E(X^2)$ **PI** **FT** their equation of the straight-line section	1.1b	A1F	$= \frac{1}{5} + \frac{157}{144}$ $= \frac{929}{720}$
Obtains the correct value of $E(X)$ **AWRT** 1.10 **PI** by correct calculation substituted into calculation to find $\text{Var}(X)$	1.1b	A1	$\text{Var}(X) = E(X^2) - (E(X))^2$ $= \frac{929}{720} - \left(\frac{79}{72}\right)^2$ $= 0.08638$ $= 0.0864$ (3sf)
Obtains the correct value of $E(X)^2$ **AWRT** 1.29 **PI** by correct calculation substituted into calculation to find $\text{Var}(X)$	1.1b	A1	
Uses the formula for the variance to obtain a calculation to find $\text{Var}(X)$	1.1a	M1	
Completes a reasoned argument to obtain the given value of $\text{Var}(X)$ A more accurate value needs to be seen before rounding, **AWRT** 0.08638	2.1	R1	
Question total		**8**	