

*Write yours and your teacher's name at the top of your answer sheets.*

# **U6 Further Mathematics Mock Paper 2 (Mechanics/Statistics)**

*Answers should be written on file paper.*

*Students – please use a new sheet of paper  
when you start the statistics section.*

**February 2025  
2024-2025**

**Duration: 1 hour 30 minutes**

**Total number of marks: 75**

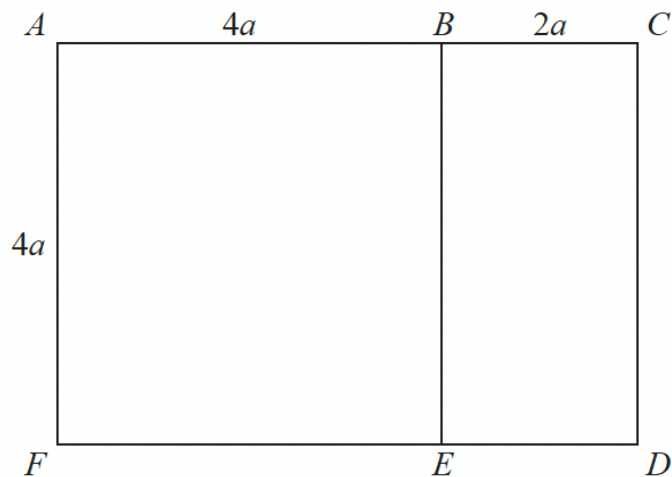
**You are permitted to use a scientific or graphical calculator in this paper.**

**Students need a formula booklet.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

# Mechanics

1.



**Figure 1**

A uniform rod of length  $24a$  is cut into seven pieces which are used to form the framework  $ABCDEF$  shown in Figure 1.

It is given that

- $AF = BE = CD = AB = FE = 4a$
- $BC = ED = 2a$
- the rods  $AF$ ,  $BE$  and  $CD$  are parallel
- the rods  $AB$ ,  $BC$ ,  $FE$  and  $ED$  are parallel
- $AF$  is perpendicular to  $AB$
- the rods all lie in the same plane

The distance of the centre of mass of the framework from  $AF$  is  $d$ .

(a) Show that  $d = \frac{19}{6}a$  (4)

(b) Find the distance of the centre of mass of the framework from  $A$ . (3)

2.

Two identical spheres,  $A$  and  $B$ , each of mass  $m$  kg, are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Just before they collide, the speeds of  $A$  and  $B$  are  $20 \text{ m s}^{-1}$  and  $10 \text{ m s}^{-1}$  respectively. The coefficient of restitution between  $A$  and  $B$  is  $e$ .

- (a) By finding, in terms of  $e$ , an expression for the velocity of  $B$  after the collision, show that the direction of motion of  $B$  is reversed by the collision. [5]

After the collision between  $A$  and  $B$ , which is **not** perfectly elastic,  $B$  goes on to collide directly with a fixed, vertical wall. The coefficient of restitution between  $B$  and the wall is  $\frac{2}{5}e$ . After the collision between  $B$  and the wall, there are no further collisions between  $A$  and  $B$ .

- (b) Determine the range of possible values of  $e$ . [7]

3.

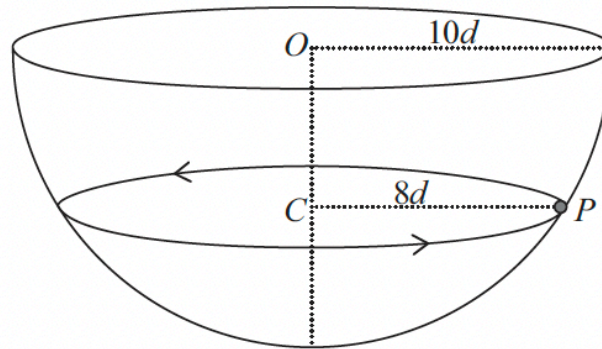


Figure 2

Figure 2 shows a hemispherical bowl of internal radius  $10d$  that is fixed with its circular rim horizontal.

The centre of the circular rim is at the point  $O$ .

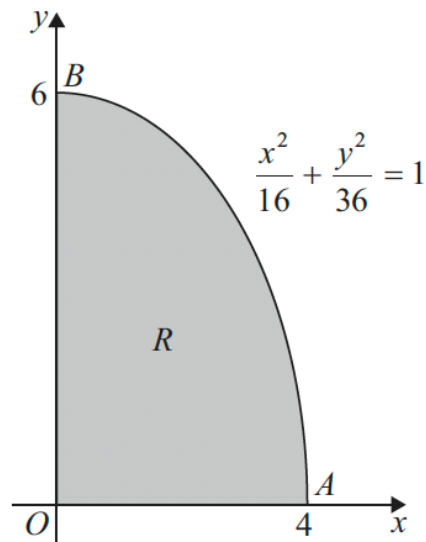
A particle  $P$  moves with **constant** angular speed on the smooth inner surface of the bowl.

The particle  $P$  moves in a horizontal circle with radius  $8d$  and centre  $C$ .

Find, in terms of  $g$ , the exact magnitude of the acceleration of  $P$ .

(6)

4.



**Figure 3**

A uniform lamina  $OAB$  is in the shape of the region  $R$ .

Region  $R$  lies in the first quadrant and is bounded by the curve with equation

$\frac{x^2}{16} + \frac{y^2}{36} = 1$ , the  $x$ -axis, and the  $y$ -axis, as shown shaded in Figure 3.

The point  $A$  is the point of intersection of the curve and the  $x$ -axis.

The point  $B$  is the point of intersection of the curve and the  $y$ -axis.

One unit on each axis represents 1 m.

The area of  $R$  is  $6\pi$

The centre of mass of  $R$  lies at the point with coordinates  $(\bar{x}, \bar{y})$

(a) Use algebraic integration to show that  $\bar{x} = \frac{16}{3\pi}$  (5)

(b) Use algebraic integration to find the exact value of  $\bar{y}$  (4)

The lamina is freely suspended from  $A$  and hangs in equilibrium with  $OA$  at angle  $\theta^\circ$  to the downward vertical.

(c) Find the value of  $\theta$  (3)

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## Statistics

5.

Sharma believes that each computer game he sells appeals equally to all age ranges. To investigate this, he takes a random sample of 100 people who play these games and asks them which of the games  $A$ ,  $B$  or  $C$  they prefer. The results are summarised in the table below.

Computer game		$A$	$B$	$C$
Age range	$< 20$	8	15	6
	$20 - 30$	21	12	9
	$> 30$	6	10	13

(a) Write down hypotheses for a suitable test to assess Sharma's belief. (1)

(b) For the test, calculate the expected frequency for

(i) those players aged under 20 who prefer game  $C$

(ii) those players aged between 20 and 30 who prefer game  $A$  (2)

(c) State the degrees of freedom of the test statistic for this test. (1)

Sharma correctly calculates the test statistic for this test to be 11.542 (to 3 decimal places).

(d) Using a 5% significance level, and stating your critical value, comment on Sharma's belief. (2)

6.

The continuous random variable  $Y$  has probability density function

$$f(y) = \begin{cases} \frac{1}{24}(y+2)(4-y) & 0 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the mode of  $Y$  is 1, justifying your reasoning. (2)

Given that  $P(Y < 1) = \frac{13}{36}$

(b) determine whether the median of  $Y$  is less than, equal to, or greater than 2. Give a reason for your answer. (2)

7.

A manager keeps a record of accidents in a canteen.

Accidents occur randomly with an average of 2.7 per month. The manager decides to model the number of accidents with a Poisson distribution.

(a) Give a reason why a Poisson distribution could be a suitable model in this situation. (1)

(b) Assuming that a Poisson model is suitable, find the probability of

(i) at least 3 accidents in the next month, (1)

(ii) no more than 10 accidents in a 3-month period, (2)

(iii) at least 2 months with no accidents in an 8-month period. (4)

8.

(a) Write down the number of ways of choosing 5 objects from 12 distinct objects. [1]

(b) Each possible set of 5 different integers selected from the integers 1, 2, ..., 12 is obtained, and for each set, the sum of the 5 integers is found. The sum  $S$  can take values between 15 and 50 inclusive. Part of the frequency distribution of  $S$  is shown in the following table, together with the cumulative frequencies.

$S$	15	16	17	18	19	20	21	22	23
Frequency	1	1	2	3	5	7	10	13	17
Cumulative Frequency	1	2	4	7	12	19	29	42	59

Use these numbers to determine the critical region for a 1-tail Wilcoxon rank-sum test at the 2% significance level when  $m = 5$  and  $n = 7$ . [2]

(c) A student says that, for a Wilcoxon rank-sum test on samples of size  $m$  and  $n$ , where  $m$  and  $n$  are large, the mean and variance of the test statistic  $R_m$  are 200 and  $616\frac{2}{3}$  respectively.

Show that at least one of these values must be incorrect. [3]

9.

A random sample of 24 adults is taken. The height,  $h$  metres, and the arm span,  $s$  metres, for each adult are recorded.

These data are summarised below.

$$S_{hh} = 0.377 \quad S_{sh} = 0.352 \quad \bar{s} = 1.70 \quad \bar{h} = 1.68$$

The least squares regression line of  $h$  on  $s$  is

$$h = a + 0.919s$$

where  $a$  is a constant.

(a) Calculate the product moment correlation coefficient.

**(3)**

A doctor uses the least squares regression line of  $h$  on  $s$  as a model to predict a person's height based on their arm span.

(b) Use the model to predict the height of an adult with arm span 1.79 metres.

**(2)**

Ewan has an arm span of 1.70 metres and a height of 1.75 metres. His information is added to the sample as the 25th adult.

(c) Explain how the gradient of the regression line for the sample of 25 adults compares with the gradient of the regression line for the original sample of 24 adults. Give a reason for your answer.

**(3)**

10.

The continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} \frac{3x}{44} + \frac{1}{22} & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find  $P(X > 2)$

**[2 marks]**

(b) Find the upper quartile of  $X$

Give your answer to two decimal places.

**[4 marks]**