U6 FM Mock (Mechanics/Statistics) 24-25 SOLUTIONS [37 + 38 = 75]

Mechanics [37]

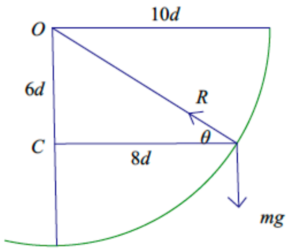
1.

1a	M(AF)	M1	2.1
	$24ad = 4a \times 4a + 4a \times 6a + 2 \times 3a \times 6a \left(= 76a^2\right)$	A1 A1	1.1b 1.1b
	$d = \dfrac{19a}{6}$ *	A1*	2.2a
		(4)	
1b	$\bar{y} = 2a$	B1	1.1b
	$D^2 = \text{their } \bar{x}^2 + \text{their } \bar{y}^2 \left(= \dfrac{361}{36}a^2 + 4a^2\right)$	M1	1.1b
	$D = \sqrt{\dfrac{505}{36}}a = \dfrac{\sqrt{505}}{6}a$	A1	1.1b
		(3)	
			(7 marks)

2.

(a)		$m \times 20 + m \times -10 = mv_A + mv_B$ $\dfrac{v_B - v_A}{20 - -10} = e$	**M1*** **M1***	3.3 3.3	**DR** Attempt at conservation of momentum Attempt at NEL	If v_A is reversed $m \times 20 + m \times -10 = -mv_A + mv_B$ $\dfrac{v_B + v_A}{20 - -10} = e$
		$m \times 20 + m \times -10 = mv_A + mv_B$ $\dfrac{v_B - v_A}{20 - -10} = e$	**A1**	1.1	Both equations correct and consistent $v_B + v_A = 10$ $v_B - v_A = 30e$	Both equations correct and consistent $v_B - v_A = 10$ $v_B + v_A = 30e$
		$2v_B = 10 + 30e$	**M1dep***	1.1	Combining two equations of correct form using simultaneous equations to find v_B	$2v_B = 10 + 30e$
		$v_B = 5 + 15e > 0$ while $u_B = -10 < 0$ *or* $v_B = 5 + 15e$ which is **positive** while original velocity of B was **negative** (so the velocity changes sign and hence direction of motion of B is reversed).	**A1** **[5]**	2.2a	Need to see clear comparison of u_b and v_b (may see $0 \le e$)	
(b)		$\dfrac{5 + 15e - v_A}{30} = e \Rightarrow v_A = \cdots$	**M1FT**	3.1b	**DR** Using either of their equations from (a) soi to give an expression for v_A consistent with their v_B. May be seen in part a.	$\dfrac{5 + 15e + v_A}{30} = e \Rightarrow v_A = \cdots$
		$v_A = 5 - 15e$ $V_B = -(2/5)e(5 + 15e)$	**A1** **B1FT**	1.1 1.1	FT their v_B from (a) (Might be positive if candidate redefines positive direction)	
		No collision => $-(2/5)(5 + 15e)e \geq 5 - 15e$	**M1**	2.2a	Correct condition for no further collision for correct v_B and v_a (condone exclusive inequality)	
		$30e^2 + 10e \leq -25 + 75e$ $30e^2 - 65e + 25 \leq 0 \Rightarrow 6e^2 - 13e + 5 \leq 0$ $(3e - 5)(2e - 1) \leq 0$	**M1**	1.1	Reduction to correct 3-term quadratic inequality WWW (condone exclusive inequality, correct direction) and using valid method to find critical values	Could be an equation to find CVs. Could be **BC**
		$\Rightarrow \frac{1}{2} \leq e \leq \frac{5}{3}$	**A1**	1.1	Condone $\frac{1}{2} < e < \frac{5}{3}$	
		So, $\frac{1}{2} \leq e < 1$	**A1** **[7]**	3.2a	Inequalities must be correct	

3.

		
Resolve vertically	M1	3.3
$R\sin\theta = mg$	A1	1.1b
Horizontal equation of motion	M1	3.3
$R\cos\theta = ma\left(= mr\omega^2\right)$	A1	1.1b
Solve for a	DM1	2.1
$\frac{g}{a} = \tan\theta \Rightarrow a = \frac{4}{3}g$	A1	1.1b

4.

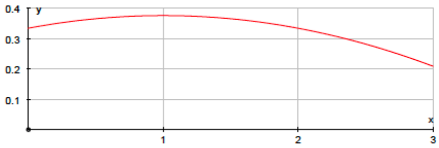
4a	Moments about the y-axis: $\int(\rho)xy\,\mathrm{d}x$	M1	3.1a
	$=(\rho)\int x\sqrt{36-\frac{9x^2}{4}}\,\mathrm{d}x = k\left(36-\frac{9x^2}{4}\right)^{\frac{3}{2}}$	M1	2.1
	$=\left[-\frac{4}{27}\left(36-\frac{9x^2}{4}\right)^{\frac{3}{2}}(\rho)\right]_0^4 \left(=32(\rho)\right)$	A1	1.1b
	$\bar{x} = \frac{\int xy\mathrm{d}x}{6\pi}$	DM1	3.1a
	$\bar{x} = \frac{32}{6\pi} = \frac{16}{3\pi}$ *	A1*	2.2a
4b	Moments about the x-axis: $\int\frac{1}{2}y^2(\rho)\mathrm{d}x\left(=\frac{1}{2}(\rho)\int 36-\frac{9x^2}{4}\mathrm{d}x\right)$	M1	3.1a
	$=\frac{1}{2}(\rho)\left[36x-\frac{3}{4}x^3\right]_0^4\left(=\frac{1}{2}(\rho)(144-48)=48(\rho)\right)$	A1	1.1b
	$\bar{y} = \frac{\int\frac{1}{2}y^2\mathrm{d}x}{6\pi}$	DM1	2.1
	$=\frac{48}{6\pi}\quad\left(=\frac{8}{\pi}\right)$	A1	2.2a
4c	Correct use of trigonometry	M1	3.1a
	$\tan\theta° = \frac{their\ \bar{y}}{4-\frac{16}{3\pi}}\left(=\frac{6}{3\pi-4}\right)$	A1ft	1.1b
	$\theta = 47.9\quad(48\text{ or better})$	A1	1.1b

Statistics [38]

5.

1(a)	H_0 : There is <u>no association</u> between <u>age</u> range and preferred <u>game</u> H_1 : There <u>is</u> an <u>association</u> between <u>age</u> range and preferred <u>game</u>	B1 (1)	1.2
(b)	(i) $\frac{29\times 28}{100}$ $= \underline{\mathbf{8.12}}$ (ii) $\frac{42\times 35}{100}$ $= \underline{\mathbf{14.7}}$	M1 A1 (2)	1.1b 1.1b
(c)	[Since (b) (i) > 5 there is no pooling so df = $(3-1)\times(3-1)=$] $\underline{\mathbf{4}}$	B1 (1)	1.1b
(d)	$\chi_4^{\,2}(0.05) = \underline{\mathbf{9.488}}$ [The test is significant so reject H_0 :] Sharma's <u>belief</u> is <u>not supported</u> / there is significant evidence of an association between <u>age</u> range and computer <u>game</u> preference.	B1ft B1 (2)	3.4/ 1.1b 2.2b
		(6 marks)	

6.

3(a)	Sketch or differentiation to find mode of Y  	$f'(y) = \frac{1}{24}(2-2y) = 0$	M1
	Mode occurs at $Y = 1$ *		A1*
			(2)
(b)	By symmetry $P(Y<2) = 2\times\frac{13}{36} = \frac{13}{18}$ (<u>or</u> 0.72 or better)		M1
	Median is less than 2 since $\frac{13}{18} > \frac{1}{2}$		A1
			(2)

7.

2(a)	Since <u>accidents</u> occur <u>randomly/independently</u> / at a <u>constant/average rate</u>	B1 (1)	2.4
(b)(i)	[A = no. of accidents in a month. $A \sim \text{Po}(2.7)$] [$P(A \geqslant 3) = 1 - P(A \leqslant 2) = 1 - 0.49362 .. = 0.50637.. =$] awrt **<u>0.506</u>**	B1 (1)	1.1b
(ii)	[T = no. of accidents in a 3-month period.] $T \sim \text{Po}(3 \times 2.7 = [8.1])$	M1	3.3
	[$P(T \leqslant 10)$] = 0.805837… = awrt **<u>0.806</u>**	A1 (2)	1.1b
(iii)	[M = no. of months with no accidents.] $M \sim B(8, e^{-2.7})$	M1	3.3
	$M \sim B(8, 0.067(2)\ldots)$	A1	1.1b
	$P(M \geqslant 2) = 1 - P(M \leqslant 1)$	M1	3.4
	= 1 – 0.903542 … = 0.096457… = awrt **<u>0.0965</u>**	A1 (4)	1.1b

8.

(a)	792	**B1** **[1]**	1.1	Allow $^{12}C_5$
(b)	0.02×792 (= 15.84) *or* 12/792 = 0.01551…, 19/792 = 0.0239…	**M1**	3.4	Find 2% of *their* **(a)**, *or* any one CF (> 1) ÷ *their* **(a)**. (*Not* 59) (18 or 20 are in tables and do not imply M1 unless clear evidence)
	0.0155 < 0.02 < 0.0239 so critical region is $(15 \leq S) \leq 19$	**A1** **[2]**	2.2a	Correct inequality from at least one correct relevant calculation. Allow "$S \leq 19$", or just "≤ 19". *Not* "CV = 19". SC: ≤ 19 with no working involving their 792: M0.
(c)	$\frac{1}{2}n(m+n+1) = 200,\ \frac{1}{12}mn(m+n+1) = 616\frac{2}{3}$	**B1**	2.1	Both correct, stated or used.
	Divide: $\frac{1}{6}n = \frac{37}{12} \Rightarrow n = 18.5 \quad (m = 12.5)$	**B1***	3.1a	Solve to get one correct answer, e.g. 7400/400, needs both previous equations but allow if one constant wrong
	$\Rightarrow n$ not an integer, hence impossible	**depB1** **[3]**	2.4	Valid reason for impossible, allow "can't be a decimal" etc, needs both previous B1s, cwo

9.

5(a)	$S_{ss} = \frac{S_{sh}}{b} = \frac{0.352}{0.919}[= 0.383\ldots]$ <u>or</u> $\frac{352}{919}$	M1
	$r = \frac{0.352}{\sqrt{0.377 \times "0.383"}}$	M1
	$r = 0.9263\ldots$ awrt **<u>0.926</u>**	A1
		(3)
(b)	$h - 1.68 = 0.919(1.79 - 1.70)$	M1
	$h = 1.76271$ awrt **<u>1.76</u>**	A1
		(2)
(c)	S_{ss} would remain the same since $s_{25} = \bar{s}$	M1
	S_{sh} would remain the same additional $(s - \bar{s})(h - \bar{h}) = 0$	M1
	So the gradient would be unchanged.	A1
		(3)

10.

6(a)	Uses $\int_2^5 \left(\frac{3x}{44}+\frac{1}{22}\right) \mathrm{d}x$ or $\int_1^2 \left(\frac{3x}{44}+\frac{1}{22}\right) \mathrm{d}x$ Condone missing $\mathrm{d}x$ **PI** by sight of **AWRT** 0.852 or 0.148	1.1a	M1	$\mathrm{P}(X>2)=\int_2^5 \frac{3x}{44}+\frac{1}{22}\,\mathrm{d}x$ $=\frac{75}{88}$
	Obtains $\frac{75}{88}$ **AWRT** 0.852	1.1b	A1	
	Subtotal		**2**	
6(b)	Integrates a multiple of $\frac{3x}{44}+\frac{1}{22}$ to an expression of the form ax^2+bx	1.1a	M1	Let q be the value of the upper quartile $\int_1^q \frac{3x}{44}+\frac{1}{22}\,\mathrm{d}x=\frac{3}{4}$ $\left[\frac{3x^2}{88}+\frac{x}{22}\right]_1^q=\frac{3}{4}$ $\frac{3q^2}{88}+\frac{q}{22}-\frac{3}{88}-\frac{1}{22}=\frac{3}{4}$ $\frac{3q^2}{88}+\frac{q}{22}-\frac{7}{88}=\frac{3}{4}$ $\frac{3q^2}{88}+\frac{q}{22}-\frac{73}{88}=0$ $q=4.31$
	Integrates $k\left(\frac{3x}{44}+\frac{1}{22}\right)$ to obtain $k\left(\frac{3x^2}{88}+\frac{x}{22}\right)$	1.1b	A1	
	Substitutes the limits q and 1 into their integral of $\frac{3x}{44}+\frac{1}{22}$, subtracts and sets equal to $\frac{3}{4}$ to form a quadratic equation in q $\frac{3q^2}{88}+\frac{q}{22}-\frac{3}{88}-\frac{1}{22}=\frac{3}{4}$ **oe** or the limits 5 and q into their integral of $\frac{3x}{44}+\frac{1}{22}$, subtracts and sets equal to $\frac{1}{4}$ to form a quadratic equation in q $\frac{75}{88}+\frac{5}{22}-\frac{3q^2}{88}-\frac{q}{22}=\frac{1}{4}$ **oe**	1.1a	M1	
	Obtains **AWRT** 4.31 If –5.64 is found it must be rejected	1.1b	A1	
	Subtotal		**4**	