U6 FM Mock Teacher X 19-20 SOLUTIONS [96]

1.

Question	Scheme	Marks	AOs
1(a)	$\mathrm{f}(x)=\mathrm{e}^{2x}\cos x \Rightarrow \mathrm{f}'(x)=2\mathrm{e}^{2x}\cos x-\mathrm{e}^{2x}\sin x$	M1	1.1a
	$\mathrm{f}''(x)=4\mathrm{e}^{2x}\cos x-2\mathrm{e}^{2x}\sin x-\left(2\mathrm{e}^{2x}\sin x+\mathrm{e}^{2x}\cos x\right)$	M1 A1	1.1b 1.1b
	$\mathrm{f}''(x)=3\mathrm{e}^{2x}\cos x-4\mathrm{e}^{2x}\sin x=p\mathrm{e}^{2x}\cos x+q\left(2\mathrm{e}^{2x}\cos x-\mathrm{e}^{2x}\sin x\right)$ $\Rightarrow p=\ldots,q=\ldots$	M1	3.1a
	$\mathrm{f}''(x)=-5\mathrm{f}(x)+4\mathrm{f}'(x)$	A1	2.1
		(5)	
(b)	$\mathrm{f}(0)=1,\mathrm{f}'(0)=2,\mathrm{f}''(0)=3,\mathrm{f}'''(0)=2,\mathrm{f}''''(0)=-7,\mathrm{f}^{v}(0)=-38$	M1	1.1b
	$\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\ldots$	M1	1.1b
	$\mathrm{f}(x)\approx 1+2x+\frac{3x^2}{2}+\frac{x^3}{3}-\frac{7x^4}{24}-\frac{19x^5}{60}$	A1	2.2a
		(3)	
			(8 marks)

2.

DR $[V=]\pi\int_0^4\left(\frac{8}{\sqrt{16+x^2}}\right)^2 \mathrm{d}x$	B1	1.1		Must be seen
$64\pi\times\frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right)$	M1	1.2		Using formulae booklet
$16\pi\tan^{-1}(1)-16\pi\tan^{-1}(0)$	M1	1.1		Substitution of correct limits must be seen
$=4\pi^2$	A1 [4]	1.1		

3.

Expresses i or z in polar form	AO1.2	B1	$\mathrm{i}=\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}$
Uses De Moivre's Theorem	AO3.1a	M1	$z=\left[\mathrm{e}^{\mathrm{i}(\frac{\pi}{2}+2n\pi)}\right]^{\frac{1}{3}}=\left[\mathrm{e}^{\mathrm{i}(\frac{\pi}{6}+\frac{2n\pi}{3})}\right]$
Finds three consecutive values for θ	AO1.1a	A1	$\frac{\pi}{6},\frac{5\pi}{6},-\frac{\pi}{2}\left(\text{or }\frac{3\pi}{2}\text{ etc}\right)$
Finds all three correct solutions for z	AO1.1b	A1	$z=\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}},\mathrm{e}^{\mathrm{i}\frac{\pi}{6}},\mathrm{e}^{\mathrm{i}\frac{5\pi}{6}}$ ALT $z=e^{i\theta}\Rightarrow z=\cos\theta+i\sin\theta$ $z^3=\mathrm{i}\Rightarrow\cos3\theta+\mathrm{i}\sin3\theta=\mathrm{i}$ $\therefore\cos3\theta=0$ and $\sin3\theta=1$ $\therefore\theta=\frac{\pi}{6},\frac{5\pi}{6},-\frac{\pi}{2}\left(\text{or }\frac{3\pi}{2}\text{ etc}\right)$ $z=\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}},\mathrm{e}^{\mathrm{i}\frac{\pi}{6}},\mathrm{e}^{\mathrm{i}\frac{5\pi}{6}}$
Total		**4**	

4.

(a)	Obtains an equation of L. Condone one error in their direction vector. Condone lack of "$\mathbf{r} =$", PI by correct $\mathbf{v}$	AO1.1a	M1	$\mathbf{r}=\begin{bmatrix}5\\-4\\6\end{bmatrix}+\mu\begin{bmatrix}1\\-2\\2\end{bmatrix}$
	Obtains a correct equation of L. Condone lack of "$\mathbf{r} =$", PI by correct $\mathbf{v}$	AO1.1b	A1	$\mathbf{v}=\begin{bmatrix}-10+\mu\\1-2\mu\\-3+2\mu\end{bmatrix}$
	Obtains their correct general vector from line to C	AO3.1a	B1F	
	Finds scalar product of their $\mathbf{v}$ and their $\overline{AB}$	AO3.1a	M1	$0=\begin{bmatrix}1\\-2\\2\end{bmatrix}\cdot\begin{bmatrix}-10+\mu\\1-2\mu\\-3+2\mu\end{bmatrix}$
	Solves to find the correct μ for their equation.	AO1.1b	A1F	$-10+\mu-2+4\mu-6+4\mu=0$ $\mu=2$
	Finds correct D	AO3.2a	A1	$D=(7,\ -8,\ 10)$
(b)	Obtains their components of $\overline{CD}$, must have their correct magnitude, but ignore sign. Allow one error.	AO1.1a	M1	$\overline{CD}=\begin{pmatrix}-8\\-3\\1\end{pmatrix}$
	Obtains their correct CD, in exact form.	AO1.1b	A1F	$CD=\sqrt{74}$
	Total		**8**	

5.

AB has direction $\begin{pmatrix}4\\2\\1\end{pmatrix}$	B1	1.1	Direction	Alternative methods possible
and any point on it is $(9+4\lambda,\ 6+2\lambda,\ 1+\lambda)$	M1	3.1a	Attempt to find point on line	
If this point lies on plane then $4(9+4\lambda)+2(6+2\lambda)+(1+\lambda)=7$ $\Rightarrow 49+21\lambda=7\Rightarrow 21\lambda=-42\Rightarrow\lambda=-2$	M1	3.1a	Attempt to find λ	
So B is where $\lambda=-4$	M1 A1	1.1a 1.1	Double λ λ soi	
$\Rightarrow B$ has coordinates $(-7,-2,-3)$	A1	3.2a		Must be coordinates
Alternatively: Distance between point and plane $=\frac{42}{\sqrt{21}}=2\sqrt{21}$ Distance between point and reflected point $=4\sqrt{21}$ **M1** Reflected point is $(x,y,z)\Rightarrow(x-9)^2+(y-6)^2+(z-1)^2$ Any point on normal line is $(9+4\lambda,6+2\lambda,1+\lambda)$ **B1** $\Rightarrow 16\lambda^2+4\lambda^2+\lambda^2=336\Rightarrow 21\lambda^2=336\Rightarrow\lambda^2=16$ **M1** $\Rightarrow\lambda=\pm4$ **A1** $(25,14,5)$ is the same side **A1** $\Rightarrow(-7,-2,-3)$	[6]			

6.

5(a)	$\int \frac{1}{\sqrt{x^2+2x+10}}\,\mathrm{d}x = \int \frac{1}{\sqrt{(x+1)^2+9}}\,\mathrm{d}x$	M1	3.1a
	$= k\sinh^{-1}\left(\frac{x+a}{b}\right)$	M1	1.1b
	$= \sinh^{-1}\left(\frac{x+1}{3}\right)(+c)$	A1	1.1b
		(3)	
(b)	$\int_2^{20} \frac{1}{\sqrt{x^2+2x+10}}\,\mathrm{d}x = \sinh^{-1}\left(\frac{20+1}{3}\right) - \sinh^{-1}\left(\frac{2+1}{3}\right)$ $= \ln\left(7+\sqrt{50}\right) - \ln\left(1+\sqrt{2}\right) = \ln\frac{7+\sqrt{50}}{1+\sqrt{2}}$	M1	1.1b
	$= \frac{1}{(20-2)}\ln\frac{7+\sqrt{50}}{1+\sqrt{2}}$	M1	2.1
	$\frac{1}{18}\ln\left(3+2\sqrt{2}\right)$ or e.g. $\frac{1}{9}\ln\left(1+\sqrt{2}\right)$	A1	2.2a
		(3)	
			(6 marks)

7.

(a)		**DR** $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$	***B1**	1.1a		Condone $2i\sin\theta = e^{i\theta} - e^{-i\theta}$
		$\sin^6\theta = \left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^6 = -\frac{1}{64}\left(e^{i\theta}-e^{-i\theta}\right)^6$	**M1**	2.1	Raising expression for $\sin\theta$ to the power 6 and $(2i)^6 = -64$.	Allow use of $\sin\theta = \frac{e^{i\theta}+e^{-i\theta}}{2i}$ for 1st two M marks only
		$(e^{i\theta}-e^{-i\theta})^6 =$ $e^{6i\theta} - 6e^{4i\theta} + 15e^{2i\theta} - 20 + 15e^{-2i\theta} - 6e^{-4i\theta} + e^{-6i\theta}$	**M1**	1.1	Genuine attempt to use binomial expansion with correct evaluated binomial coefficients. Condone sign errors	If i omitted from denominator their expression for sin θ then only this M mark can still be awarded
		$e^{6i\theta}+e^{-6i\theta} - 6\left(e^{4i\theta}+e^{-4i\theta}\right) + 15\left(e^{2i\theta}+e^{-2i\theta}\right) - 20$ $= 2\cos 6\theta - 6\times 2\cos 4\theta + 15\times 2\cos 2\theta - 20$	**M1**	2.1	Collecting terms and <u>using</u> $e^{i\phi}+e^{-i\phi} = 2\cos\phi$ at least once.	
		$\therefore \sin^6\theta =$ $-\frac{1}{64}(2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20)$ $= \frac{1}{32}(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta)$	**dep*A1**	1.1	AG. Fully correct argument	
			[5]			
(b)		**DR** $\theta = \frac{\pi}{8}$ and eg $\cos 2\theta = \frac{\sqrt{2}}{2}$	***M1**	2.1	Choice of θ soi and calculation of at least one cos term	
		$\sin^6\frac{\pi}{8} = \frac{1}{32}\left(10 - 15\times\frac{\sqrt{2}}{2} - \frac{-\sqrt{2}}{2}(+6(0))\right)$	**dep*M1**	1.1	Substitution and calculation of all cos terms	Terms must be shown distinct either in this line or in the form of $\cos n\frac{\pi}{8}$
		$\sin\frac{\pi}{8} = \sqrt[6]{\frac{1}{64}\left(20 - 15\sqrt{2} + \sqrt{2}\right)}$ $= \frac{1}{2}\sqrt[6]{20 - 14\sqrt{2}}$	**A1**	2.2a	AG Some intermediate working must be seen	
			[3]			

8.

(a)	Uses correct expressions for $\cosh x$ and $\sinh x$,and uses them to simplify LHS.	AO1.1a	M1	$\cosh^3 x = \frac{1}{8}\left(e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}\right)$
	Finds a correct, unsimplified expansion of the LHS in terms of exponentials.	AO1.1b	A1	$\sinh^3 x = \frac{1}{8}\left(e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}\right)$
	Completes a rigorous argument to obtain the correct result. Must include clear definitions for $\cosh x$ & $\sinh x$. NMS = 0/3	AO2.1	R1	$\cosh^3 x + \sinh^3 x = \frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}$
(b)	Finds $\cosh^3 x - \sinh^3 x$ in exponential form PI correct exponential expression.	AO3.1a	B1	$\cosh^6 x - \sinh^6 x = \left(\cosh^3 x + \sinh^3 x\right)\left(\cosh^3 x - \sinh^3 x\right)$ $\cosh^3 x - \sinh^3 x = \frac{3}{4}e^{x} + \frac{1}{4}e^{-3x}$
	Uses their expressions to find $\cosh^6 x - \sinh^6 x$ in exponential form.	AO3.1a	M1	$\cosh^6 x - \sinh^6 x = \left(\frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}\right)\left(\frac{3}{4}e^{x} + \frac{1}{4}e^{-3x}\right)$
	Obtains their correct result	AO1.1b	A1F	
	Correctly separates out $\frac{3}{8}\cosh 4x$ or equivalent from their expression	AO2.2a	M1	$= \frac{3}{16}e^{4x} + \frac{9}{16} + \frac{1}{16} + \frac{3}{16}e^{-4x}$ $= \frac{3}{8}\left(\frac{e^{4x} + e^{-4x}}{2}\right) + \frac{5}{8}$ $= \frac{3\cosh 4x + 5}{8}$
	Completes a rigorous argument to obtain the correct result.	AO2.1	R1	
	Total		**8**	

9.

(a)		$=\frac{1}{1+\left(\sqrt{2x}\right)^2}\times\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{2x}\right)$ **oe** $=\frac{1}{1+2x}\times\frac{\sqrt{2}}{2\sqrt{x}}=\frac{1}{1+2x}\times\frac{1}{\sqrt{2x}}$	M1 A1 [2]	1.1a 1.1	Attempt to differentiate using chain rule	i.e. product of 2 terms
		Alternatively: $\tan y=\sqrt{2x}$ $\Rightarrow \sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{2}}{2\sqrt{x}}\Rightarrow\left(1+\tan^2 x\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{2}}{2\sqrt{x}}$ **M1** $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+2x}\times\frac{\sqrt{2}}{2\sqrt{x}}$ **A1**	[4]		Make a substitution	

(b)		$\int_{1/6}^{1/2}\frac{1}{(1+2x)\sqrt{x}}\,\mathrm{d}x=\sqrt{2}\int_{1/6}^{1/2}\frac{1}{(1+2x)\sqrt{2x}}\,\mathrm{d}x$ $=\sqrt{2}\left[\tan^{-1}\sqrt{2x}\right]_{1/6}^{1/2}=\sqrt{2}\left(\tan^{-1}1-\tan^{-1}\frac{1}{\sqrt{3}}\right)$ $=\sqrt{2}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\frac{\sqrt{2}}{12}\pi$ So $k=\frac{\sqrt{2}}{12}$	M1 A1 M1 A1	3.1a 1.1 1.1 1.1	Get into form of (a). Ignore limits Correct form Use (a) and correct limits in correct order. oe
		Alternatively: Let $u=\sqrt{x}$ **M1** $\mathrm{d}u=\frac{1}{2\sqrt{x}}\mathrm{d}x\Rightarrow \mathrm{d}x=2\sqrt{x}\mathrm{d}u=2u\mathrm{d}u$ $\int_{\frac{1}{6}}^{\frac{1}{2}}\frac{\sqrt{x}}{\left(x+2x^2\right)}\mathrm{d}x=\int_{u=\frac{1}{6}}^{u=\frac{1}{2}}\frac{u}{\left(u^2+2u^4\right)}2u\mathrm{d}u=2\int_{u=\frac{1}{6}}^{u=\frac{1}{2}}\frac{1}{\left(1+2u^2\right)}\mathrm{d}u$ **A1** $=\sqrt{2}\left[\tan^{-1}u\sqrt{2}\right]_{u=\frac{1}{6}}^{u=\frac{1}{2}}$ **M1** $=\sqrt{2}\left[\tan^{-1}\sqrt{2x}\right]_{u=\frac{1}{6}}^{u=\frac{1}{2}}=\sqrt{2}\left(\tan^{-1}1-\tan^{-1}\frac{1}{\sqrt{3}}\right)=\sqrt{2}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ $=\frac{\pi\sqrt{2}}{12}$ **A1**	[4]		Make a substitution Get into correct form Use standard result with correct limits in correct order

10.

Divides through by x	AO1.1a	M1	$\frac{dy}{dx} - \frac{2y}{x} = \frac{x^2}{\sqrt{4-2x-x^2}}$
Recognises that the Integrating Factor Method can be applied and finds correct integrating factor, accept $e^{-2\ln x}$	AO3.1a	M1	$\int P dx = -\int \frac{2}{x} dx = -2\ln x$
Multiplies equation by their integrating factor	AO1.1a	M1	Integrating factor $= e^{\int P dx} = e^{-2\ln x} = x^{-2}$
Integrates LHS to obtain $\frac{y}{x^2}$	AO1.1b	A1	$\frac{1}{x^2}\frac{dy}{dx} - \frac{2y}{x^3} = \frac{1}{\sqrt{4-2x-x^2}}$
Recognises the need to complete the square inside the square root.	AO3.1a	M1	$\frac{d}{dx}\left(\frac{y}{x^2}\right) = \frac{1}{\sqrt{4-2x-x^2}}$
Correctly uses the appropriate inverse sine, inverse cosh or inverse sinh function to integrate all (or part of) their RHS.	AO3.1a	M1	
Finds correct solution including constant of integration. ACF Accept $\frac{y}{x^2} = \ldots$	AO1.1b	A1	To find $\int \frac{1}{\sqrt{4-2x-x^2}} dx$ $4-2x-x^2 = 5-(x+1)^2$ $\int \frac{1}{\sqrt{4-2x-x^2}} dx = \int \frac{1}{\sqrt{5-(x+1)^2}} dx$ $\therefore y = x^2\left\{\sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c\right\}$
Total		**7**	

11.

(a)	Forms a differential equation for the foxes.	AO3.3	B1	$\frac{dy}{dt} \propto x \Rightarrow \frac{dy}{dt} = kx$
	Forms a differential equation for the rabbits.	AO3.3	B1	$\frac{dy}{dt} = 20, x = 200 \Rightarrow k = 0.1$
	Differentiates 'their' equation that contains y and obtains expression with at least two terms correct.	AO1.1a	M1	$\frac{dy}{dt} = 0.1x$ $\frac{dx}{dt} = 1.2x - 1.1y$
	Formulates a second order linear differential equation.	AO3.1a	M1	$y = \frac{1.2x}{1.1} - \frac{1}{1.1}\frac{dx}{dy}$
	Obtains roots of auxiliary equation for 'their' second order differential equation.	AO1.1a	M1	$\frac{dy}{dt} = \frac{1.2}{1.1}\frac{dx}{dt} - \frac{1}{1.1}\frac{d^2x}{dt^2}$
	States correct general solution. FT provided all M marks have been awarded	AO1.1b	A1F	$0.1x = \frac{1.2}{1.1}\frac{dx}{dt} - \frac{1}{1.1}\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} - 1.2\frac{dx}{dt} + 0.11x = 0$
	Uses initial population to find equation linking constants for their general solution.	AO3.4	M1	$\lambda^2 - 1.2\lambda + 0.11 = 0$ $\lambda = 0.1$ or 1.1 $x = Ae^{0.1t} + Be^{1.1t}$
	Obtains initial rate of change for rabbits from 'their' DE.	AO3.4	M1	$t = 0, x = 80$ $A + B = 80$
	Differentiates and obtains a second equation for A and B from 'their' general solution.	AO1.1a	M1	$t = 0, x = 74$ $74 = 0.1A + 1.1B$ $74 = 0.1(80 - B) + 1.1B$ $B = 68, A = 12$
(a)(ii)	Substitutes 0.7 and obtains approximately 160. CAO	AO3.4	A1	$12e^{0.07} + 68e^{0.77} = 159.7$
(b)	States a suitable refinement about the fact that an increased rabbit population will require more food supply or other valid refinement	AO3.5c	B1	Take account of the food available for the rabbits as this may limit population growth.
	Total		**11**	

12.

4(a) Way 1	$C+\text{i}S=\cos\theta+\text{i}\sin\theta+\frac{1}{2}(\cos 5\theta+\text{i}\sin 5\theta)\left(+\frac{1}{4}(\cos 9\theta+\text{i}\sin 9\theta)+\ldots\right)$	M1	1.1b
	$=\text{e}^{\text{i}\theta}+\frac{1}{2}\text{e}^{5\text{i}\theta}\left(+\frac{1}{4}\text{e}^{9\text{i}\theta}+\ldots\right)$	A1	2.1
	$C+\text{i}S=\frac{\text{e}^{\text{i}\theta}}{1-\frac{1}{2}\text{e}^{4\text{i}\theta}}$	M1	3.1a
	$=\frac{2\text{e}^{\text{i}\theta}}{2-\text{e}^{4\text{i}\theta}}$ *	A1*	1.1b
		(4)	
(a) Way 2	$C+\text{i}S=\cos\theta+\text{i}\sin\theta+\frac{1}{2}(\cos 5\theta+\text{i}\sin 5\theta)\left(+\frac{1}{4}(\cos 9\theta+\text{i}\sin 9\theta)+\ldots\right)$	M1	1.1b
	$C+\text{i}S=\cos\theta+\text{i}\sin\theta+\frac{1}{2}(\cos\theta+\text{i}\sin\theta)^5\left(+\frac{1}{4}(\cos\theta+\text{i}\sin\theta)^9+\ldots\right)$	A1	2.1
	$C+\text{i}S=\frac{\cos\theta+\text{i}\sin\theta}{1-\frac{1}{2}(\cos\theta+\text{i}\sin\theta)^4}=\frac{\text{e}^{\text{i}\theta}}{1-\frac{1}{2}\text{e}^{4\text{i}\theta}}$	M1	3.1a
	$=\frac{2\text{e}^{\text{i}\theta}}{2-\text{e}^{4\text{i}\theta}}$ *	A1*	1.1b
		(4)	
(b) Way 1	$\frac{2\text{e}^{\text{i}\theta}}{2-\text{e}^{4\text{i}\theta}}\times\frac{2-\text{e}^{-4\text{i}\theta}}{2-\text{e}^{-4\text{i}\theta}}$	M1	3.1a
	$\frac{4\text{e}^{\text{i}\theta}-2\text{e}^{-3\text{i}\theta}}{4-2\text{e}^{-4\text{i}\theta}-2\text{e}^{4\text{i}\theta}+1}$	A1	1.1b
	$\frac{4\cos\theta+4\text{i}\sin\theta-2\cos 3\theta+2\text{i}\sin 3\theta}{5-2\cos 4\theta+2\text{i}\sin 4\theta-2\cos 4\theta-2\text{i}\sin 4\theta}$ **Dependent on the first M**	dM1	2.1
	$S=\frac{4\sin\theta+2\sin 3\theta}{5-4\cos 4\theta}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\frac{2\text{e}^{\text{i}\theta}}{2-\text{e}^{4\text{i}\theta}}=\frac{2(\cos\theta+\text{i}\sin\theta)}{2-(\cos 4\theta+\text{i}\sin 4\theta)}\times\frac{2-(\cos 4\theta-\text{i}\sin 4\theta)}{2-(\cos 4\theta-\text{i}\sin 4\theta)}$	M1	3.1a
	$\frac{4\cos\theta+4\text{i}\sin\theta-2\cos\theta\cos 4\theta-2\sin\theta\sin 4\theta+2\text{i}\sin 4\theta\cos\theta-2\text{i}\sin\theta\cos 4\theta}{4+\cos^2 4\theta+\sin^2 4\theta-4\cos 4\theta}$	A1	1.1b
	$\frac{4\cos\theta+4\text{i}\sin\theta-2\cos 3\theta+2\text{i}\sin 3\theta}{5-2\cos 4\theta+2\text{i}\sin 4\theta-2\cos 4\theta-2\text{i}\sin 4\theta}$ **Dependent on the first M**	dM1	2.1
	$S=\frac{4\sin\theta+2\sin 3\theta}{5-4\cos 4\theta}$ *	A1*	1.1b
			(8 marks)

13.

(a)	Models the motion of the particle by forming a second order differential equation. (must have correct terms but allow sign errors	AO3.3	M1	$M\frac{d^2x}{dt^2} = -4M\frac{dx}{dt} - 8Mx$ $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0$
	Obtains correct differential equation	AO1.1b	A1	$\lambda^2 + 4\lambda + 8 = 0$ $\lambda = -2 \pm 2i$
	Forms and solves auxiliary equation for 'their' D.E.	AO1.1a	M1	Complex roots ⇒ General solution is of the form: $x = Ae^{-2t}\cos(2t + B)$
	States a correct form of the general solution for 'their' auxiliary solution. (ft only if both M1 marks have been awarded)	AO1.1b	A1F	$\dot{x} = -2Ae^{-2t}\cos(2t + B) - 2Ae^{-2t}\sin(2t + B)$ $\dot{x}(0) = 0$ so,
	Uses initial conditions to find arbitrary constants for 'their' solution	AO1.1a	M1	$-2A\cos(B) - 2A\sin(B) = 0$ $\Rightarrow \tan B = -1$ $\Rightarrow B = -\frac{\pi}{4}$ $x(0) = 1$
	Obtains correct value for one of 'their' constants (ft only if all M1 marks have been awarded)	AO1.1b	A1F	$\Rightarrow \frac{A}{\sqrt{2}} = 1$ $\Rightarrow A = \sqrt{2}$
	Obtains correct value for both of 'their' constants (ft only if all M1 marks have been awarded)	AO1.1b	A1F	$\therefore x = \sqrt{2}e^{-2t}\cos(2t - \frac{\pi}{4})$
	Uses 'their' model to describe the motion of the particle either as a written description or shown on a clearly labelled graph.	AO3.4	A1F	∴ the particle oscillates about O, with period π seconds and decreasing amplitude.

(b)	Refines their DE model to account for altered resistive force by introducing a new coefficient for $\frac{\mathrm{d}x}{\mathrm{d}t}$	AO3.5c	B1	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\alpha\frac{\mathrm{d}x}{\mathrm{d}t}+8x=0$ Critical damping $\Rightarrow$
	Uses or states condition for critical damping	AO1.2	B1	$\lambda^2+\alpha\lambda+8=0$ must have equal roots
	Deduces value for coefficient of $\frac{\mathrm{d}x}{\mathrm{d}t}$	AO2.2a	R1	$\alpha^2=32$ $\alpha=4\sqrt{2}$ Resistive force should have magnitude $4\sqrt{2}Mv$
	States resistive force	AO3.4	B1	
	Total		**12**	