U6 FM Mock Teacher X 24-25 SOLUTIONS [100]

1.

$\dfrac{1}{\sqrt{(1-(x^2)^2}}$	B1	1.1	For $\dfrac{1}{\sqrt{(1-(x^2)^2}}$ seen.
$\times 2x$	M1	1.1	For $2x \times \mathrm{f}(x)$ where $\mathrm{f}(x)=\dfrac{1}{\sqrt{1-(x^2)^2}}$ or $\dfrac{1}{\sqrt{1-x^2}}$ ONLY.
$\left(\dfrac{dy}{dx}=\right)\dfrac{2x}{\sqrt{1-x^4}}$	A1	1.1	Allow any equivalent correct form e.g. $2x(1-x^4)^{-0.5}$. Must be in terms of x. Condone $(x^2)^2$ for x^4. ISW once correct answer seen.
	[3]		
Alternative method			
$\cos y \dfrac{dy}{dx}=2x$ or $\cos y = 2x\dfrac{dx}{dy}$	B1		For correctly differentiating implicitly with respect to either x or y.
$\sqrt{1-(x^2)^2}\dfrac{dy}{dx}=2x$	M1		Replacing $\cos y$ with $\pm\sqrt{\pm 1 \pm (x^2)^2}$ in their derivative of the form $\pm\cos y\dfrac{dy}{dx}=2x$ (or equivalent if differentiating with respect to y).
$\left(\dfrac{dy}{dx}=\right)\dfrac{2x}{\sqrt{1-x^4}}$	A1		Allow any equivalent correct form e.g. $2x(1-x^4)^{-0.5}$. Must be in terms of x. Condone $(x^2)^2$ for x^4. ISW once correct answer seen.
	[3]		

2.

7(a)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(2\mathbf{i}+\mathbf{j}-4\mathbf{k})=6-10+4=0$ $(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=18-10-8=0$	M1	1.1b
	So $3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ is perpendicular to Π	A1	2.1
		(2)	
(b)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(\mathbf{i}-2\mathbf{j}+3\mathbf{k})=3+20-3=20$	M1	1.1b
	$3x-10y-z=20$	A1	2.5
		(2)	
(c)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(5\mathbf{i}+p\mathbf{j}-7\mathbf{k})=\text{"20"}$ $\Rightarrow 15-10p+7=\text{"20"}\Rightarrow p=\ldots$	M1	3.1a
	$p=0.2$ (oe)	A1	1.1b
		(2)	
(d)	E.g. $1+2\lambda=5+6\mu,\ 3-4\lambda=-7+8\mu \Rightarrow \lambda=\ldots$ or $\mu=\ldots$ $\mu=0.1$ (or $\lambda=2.3$) $\Rightarrow A(5.6,\ 0.3,\ -6.2)$	M1	1.1b
	$12\mathbf{i}-11\mathbf{j}+6\mathbf{k}-(5.6\mathbf{i}+0.3\mathbf{j}-6.2\mathbf{k})=6.4\mathbf{i}-11.3\mathbf{j}+12.2\mathbf{k}$ $(6.4\mathbf{i}-11.3\mathbf{j}+12.2\mathbf{k})\cdot(3\mathbf{i}-10\mathbf{j}-\mathbf{k})=19.2+113-12.2=120$	M1	3.1a
	$120=\sqrt{6.4^2+11.3^2+12.2^2}\sqrt{3^2+10^2+1^2}\cos\alpha \Rightarrow \alpha=\ldots$ or e.g. $120=\sqrt{6.4^2+11.3^2+12.2^2}\sqrt{3^2+10^2+1^2}\sin\alpha \Rightarrow \alpha=\ldots$	M1	1.1b
	Angle between AB and plane $\theta=40°$ (awrt)	A1	1.1b

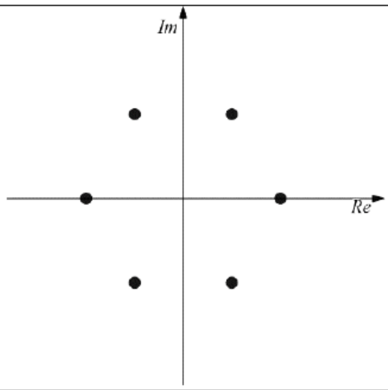
3.

DR			
$\int \frac{18}{x^2\sqrt{x}}\,dx = 18\left(-\frac{2}{3}x^{-\frac{3}{2}}\right)(+c)$	**M1***	**1.1**	For obtaining $ax^{-\frac{3}{2}}$ where $a \neq 0$.
$= 18\lim_{k\to\infty}\left(-\frac{2}{3}k^{-\frac{3}{2}} - \left(-\frac{2}{3}\times 9^{-\frac{3}{2}}\right)\right)$	**M1**	**1.1**	Correct use of 9 as a lower limit and any letter (except x) for the upper limit (so must be considering a finite upper limit) in their integrated expression (indicated by their power increased by 1). Need not see mention of limiting process for this mark.
$k^{-\frac{3}{2}} \to 0$ as $k \to \infty$	**B1dep***	**2.1**	Taking limit as $k \to \infty$ for their expression of the form $ax^{-\frac{3}{2}}$ (so $\frac{1}{\sqrt{\infty^3}} = 0$ oe is **B0**). Implied by, for example, $\lim_{k\to\infty}\left[-\frac{2}{3}k^{-\frac{3}{2}} - \ldots\right] = 0 - \ldots$ but not, for example, for $-\frac{2}{3}k^{-\frac{3}{2}} - \ldots = 0 - \ldots$ without clear use of limiting process.
$= \frac{4}{9}$	**A1**	**2.2a**	cao from correct integrated expression and finite upper limit (so dependent on both previous **M** marks but not the **B** mark). Accept equivalent **exact** forms e.g. $\frac{12}{27}$.
	[4]		

4.

1(a)	$4\sinh^3 x + 3\sinh x \equiv 4\left(\frac{e^x - e^{-x}}{2}\right)^3 + 3\left(\frac{e^x - e^{-x}}{2}\right)$ $\equiv 4\left(\frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8}\right) + 3\left(\frac{e^x - e^{-x}}{2}\right)$	M1	2.1
	$\equiv \frac{e^{3x} - e^{-3x}}{2} \equiv \sinh 3x$ *	A1*	1.1b
		(2)	
(b)	$\sinh 3x = 19\sinh x \Rightarrow 4\sinh^3 x + 3\sinh x = 19\sinh x$ $\sinh 3x = 19\sinh x \Rightarrow 4\sinh^3 x - 16\sinh x = 0$ $4\sinh x(\sinh^2 x - 4) = 0$	M1	3.1a
	$\sinh x = 0 \Rightarrow x = 0$	B1	2.2a
	$\sinh^2 x = 4 \Rightarrow \sinh x = \pm 2$ $\Rightarrow x = \ln\left(\pm 2 + \sqrt{(\pm 2)^2 + 1}\right)$	M1	1.1b
	$x = \ln\left(2+\sqrt{5}\right)$ **or** $x = \ln\left(-2+\sqrt{5}\right)$ oe e.g. $x = -\ln\left(2+\sqrt{5}\right)$	A1	1.1b
	$x = \ln\left(2+\sqrt{5}\right)$ **and** $x = \ln\left(-2+\sqrt{5}\right)$ Alternatively, $x = \ln\left(\sqrt{5}\pm 2\right)$ oe e.g. $x = \pm\ln\left(2+\sqrt{5}\right)$ or $\frac{1}{2}\ln\left(9 \pm 4\sqrt{5}\right)$	A1	1.1b
		(5)	
			(7 marks)

5.

Question	Scheme	Marks	AOs
7(a)	$z = \mathrm{e}^{\frac{k\pi}{3}\mathrm{i}}, \quad k = 0, 1, 2, 3, 4, 5$	M1 A1	1.1b 1.1b
		(2)	
(b)		B1 dB1	2.2a 1.1b
		(2)	
(c)	e.g. $\left(\sqrt{3}+\mathrm{i}\right)^6 = \left(2\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}\right)^6 = 64\mathrm{e}^{\mathrm{i}\pi} = -64$ * **or** $\left[2\left(\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}\right)\right]^6 = 2^6(\cos\pi+\mathrm{i}\sin\pi) = 64(-1) = -64$ * **or** $\left(\sqrt{3}+\mathrm{i}\right)^6 = \left(\sqrt{3}\right)^6 + 6\left(\sqrt{3}\right)^5\mathrm{i} - 15\left(\sqrt{3}\right)^4 - 20\left(\sqrt{3}\right)^3\mathrm{i} + 15\left(\sqrt{3}\right)^2 + 6\sqrt{3}\mathrm{i} + \mathrm{i}^6$ $= 27 - 135 + 45 - 1 = -64$ * **or** $\left(\sqrt{3}+\mathrm{i}\right)^6 = = 27 + 54\sqrt{3}\mathrm{i} + 135\mathrm{i}^2 + 60\sqrt{3}\mathrm{i}^3 + 45\mathrm{i}^4 + 6\sqrt{3}\mathrm{i}^5 + \mathrm{i}^6$ $= 27 + 54\sqrt{3}\mathrm{i} - 135 - 60\sqrt{3}\mathrm{i} + 45 + 6\sqrt{3}\mathrm{i} - 1 = -64$ *	M1 A1*	1.1b 2.1
		(2)	
(d)	$r = 2$	B1	2.2a
	$z = 2\mathrm{e}^{\frac{\pi}{6}\mathrm{i}} \times \mathrm{e}^{\frac{k\pi}{3}\mathrm{i}}, \quad k = 0, 1, 2, 3, 4, 5$	M1	3.1a
	$z = 2\mathrm{e}^{\left(\frac{\pi}{6}+\frac{k\pi}{3}\right)\mathrm{i}}, \quad k = 0, 1, 2, 3, 4, 5$	A1	1.1b
		(3)	
			(9 marks)

6.

13(a)	Expands $(\cos\theta + i\sin\theta)^3$ PI correct real part	1.1a	M1	$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$ $= \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$ Equating real parts $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$ $= \cos^3\theta - 3\cos\theta(1 - \cos^2\theta)$ $= 4\cos^3\theta - 3\cos\theta$
	Equates real parts and uses $\sin^2\theta = 1 - \cos^2\theta$	1.1a	M1	
	Completes a reasoned argument using de Moivre's theorem to show $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ AG	2.1	R1	
13(b)	Equates imaginary parts and uses $\cos^2\theta = 1 - \sin^2\theta$	1.1a	M1	Equating imaginary parts $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$ $= 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$
	Obtains $3\sin\theta - 4\sin^3\theta$	1.1b	A1	
13(c)	Substitutes expressions for $\cos 3\theta$ and their $\sin 3\theta$ into $\cot 3\theta = \dfrac{\cos 3\theta}{\sin 3\theta}$ or $\tan 3\theta = \dfrac{\sin 3\theta}{\cos 3\theta}$	1.1b	B1F	$\cot 3\theta = \dfrac{\cos 3\theta}{\sin 3\theta} = \dfrac{4\cos^3\theta - 3\cos\theta}{3\sin\theta - 4\sin^3\theta}$ $= \dfrac{4\left(\cos^3\theta / \sin^3\theta\right) - 3\left(\cos\theta / \sin^3\theta\right)}{3\left(\sin\theta / \sin^3\theta\right) - 4\left(\sin^3\theta / \sin^3\theta\right)}$ $= \dfrac{4\cot^3\theta - 3\cot\theta\operatorname{cosec}^2\theta}{3\operatorname{cosec}^2\theta - 4}$ $= \dfrac{4\cot^3\theta - 3\cot\theta\left(1+\cot^2\theta\right)}{3\left(1+\cot^2\theta\right) - 4}$ $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$
	Manipulates their rational function of $\sin\theta$ and $\cos\theta$ to obtain at least one instance of $\cot\theta$ or $\tan\theta$	3.1a	M1	
	Manipulates their rational function of $\sin\theta$ and $\cos\theta$ to obtain only $\cot\theta$ (and $\operatorname{cosec}\theta$) terms	3.1a	M1	
	Completes a reasoned argument from the final or intermediate results in parts **(a)** and **(b)** to show $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$ AG	2.1	R1	

7.

(a)	$F = ma$ so So $-(15\sin 4t + 6v\tan 2t) = 3\frac{dv}{dt}$	M1	3.3	Using $F = 3a$ and $a = \frac{dv}{dt}$ to form a differential equation - allow minor slips or sign errors but intention must be clear. Their F must be two terms only.
	$\frac{dv}{dt} + 2v\tan 2t = -5\sin 4t$ (so $P(t) = 2\tan 2t$ and $Q(t) = -5\sin 4t$)	A1	1.1	The correct differential equation in the form $\frac{dv}{dt} + P(t)v = Q(t)$ can imply this mark. $P(t)$ and $Q(t)$ do not need to be explicitly stated. ISW once correct form seen. If not written in the form, $\frac{dv}{dt} + P(t)v = Q(t)$ then $P(t)$ and $Q(t)$ must be explicitly stated.
(b)	$\mathrm{I}(t) = e^{\int 2\tan 2t\, dt}$	M1*	1.1	For $\mathrm{I}(t) = e^{\int P(t)dt}$ for their $P(t)$
	$= e^{-\ln(\cos 2t)}$ $(= \sec 2t)$	M1	1.1	For $\mathrm{I}(t) = e^{\pm k\ln(\cos 2t)}$ or $e^{\pm k\ln(\sec 2t)}$ or $\pm k\cos 2t$ or $\pm k\sec 2t$ or $\pm a\cos^k 2t$ or $\pm a\sec^k 2t$ for $a, k \neq 0$.
	$\left(\frac{dv}{dt} + 2v\tan 2t\right)\times \mathrm{I}(t) = -5\sin 4t \times \mathrm{I}(t) \Rightarrow \frac{d}{dt}(v\times \mathrm{I}(t)) = -5\sin 4t \times \mathrm{I}(t)$			
	$v\times\sec 2t = -5\int(\sin 4t \times \sec 2t)dt$	M1dep*	1.1	For $v\times \mathrm{I}(t) = k_1\int \sin 4t \times \mathrm{I}(t)\,dt$ with their $\mathrm{I}(t)$ (in **any** form) and $k_1 \neq 0$.
	$(v\sec 2t =) -10\int \frac{\sin 2t\cos 2t}{\cos 2t}dt = -10\int \sin 2t\, dt$	M1	1.1	For simplifying *RHS* to $k_2\int \sin 2t\, dt$ for any $k_2 \neq 0$ - dependent on all previous **M** marks.
	$v\sec 2t = 5\cos 2t\ (+c)$	A1	1.1	For correct general solution (any equivalent form). Condone lack of $+c$.
	$v(0) = 4.5 \Rightarrow c = -0.5$	M1	3.3	Using $v(0) = 4.5$ to find constant term – dependent on first three **M** marks and an attempt at integration.
	$0 = 5\cos 2t - 0.5 \Rightarrow \cos 2t = 0.1$	M1dep*	3.4	For a two-term equation of the form $\cos 2t = k_3$ where $\lvert k_3\rvert < 1$ and $k_3 \neq 0$ - dependent on all previous **M** marks.
	So, B stationary after 0.735 seconds.	A1	2.2a	Ignore if any other solution(s) found. Allow awrt 0.735 – for reference 0.7353144… 42.1 seconds (calculated in degrees) scores **A0**.

8.

$\frac{12x^3}{(2x+1)(2x^2+1)} = A + \frac{B}{2x+1} + \frac{Cx+D}{2x^2+1}$	B1	1.1	Correct form (possibly implied by correct identity).
$12x^3 \equiv A(2x+1)(2x^2+1) + B(2x^2+1) + (Cx+D)(2x+1)$	M1*	1.1	Identity without fractions. Follow through their partial fraction expression with one denominator of $2x+1$ and the other with $2x^2+1$ - both numerators must contain at least a constant unknown. Some examples for **M1** below: $\frac{B}{2x+1} + \frac{Cx+D}{2x^2+1}$ so $12x^3 \equiv B(2x^2+1) + (Cx+D)(2x+1)$ $A + \frac{B}{2x+1} + \frac{C}{2x^2+1}$ so $12x^3 \equiv A(2x+1)(2x^2+1) + B(2x^2+1) + C(2x+1)$ $\frac{B}{2x+1} + \frac{C}{2x^2+1}$ so $12x^3 \equiv B(2x^2+1) + C(2x+1)$. $\frac{A}{2x+1} + \frac{Bx^2+Cx+D}{2x^2+1}$ so $12x^3 \equiv A(2x^2+1) + (Bx^2+Cx+D)(2x+1)$.
For example: Equating coefficients of $x^3 : A = 3$ Let $x = 0$, gives $A + B + D = 0$	M1dep*	1.1	Equates coefficients or substitutes to find an equation involving only their unknowns. Do not award this mark if only two unknowns in their partial fractions (so must be at least three unknowns (or implied unknowns)).
	A1	1.1	Any two ($A = 3$, $B = -1$, $C = -2$, $D = -2$) unknowns correct from a correct partial fraction form.
$3 - \frac{1}{2x+1} - \frac{2x+2}{2x^2+1}$	A1	1.1	All four unknowns correct – condone stating the correct form of the partial fractions anywhere together with all four unknowns correctly stated without necessarily bringing both parts together as a single expression at the end.

9.

8(a)	$\frac{d^2x}{dt^2}=\frac{dx}{dt}+\frac{dy}{dt}=\frac{dx}{dt}+3y-2x$	M1	1.1b
	$=\frac{dx}{dt}+3\left(\frac{dx}{dt}-x\right)-2x$	M1	2.1
	$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0*$	A1*	1.1b
		(3)	
(b)	$m^2-4m+5=0\Rightarrow m=...$	M1	3.4
	$m=2\pm i$	A1	1.1b
	$x=e^{\alpha t}\left(A\cos\beta t+B\sin\beta t\right)$	M1	3.4
	$x=e^{2t}\left(A\cos t+B\sin t\right)$	A1	1.1b
		(4)	
(c)	$y=\frac{dx}{dt}-x=e^{2t}\left(B\cos t-A\sin t+2A\cos t+2B\sin t\right)-e^{2t}\left(A\cos t+B\sin t\right)$	M1	3.4
	$y=e^{2t}\left(\left(A+B\right)\cos t+\left(B-A\right)\sin t\right)$	A1	1.1b
		(2)	
(d)	$A=100,\ 275=A+B\Rightarrow B=175$	M1	3.3
	$x=y\Rightarrow 100\cos t+175\sin t=275\cos t+75\sin t\Rightarrow \tan t=...$	dM1	3.1a
	$\tan t=1.75$	A1	1.1b
	$T=24\tan^{-1}\left(1.75\right)=...$	M1	3.2a
	$=25.24$	A1	1.1b
		(5)	
(e)	E.g. • Both populations become negative for some times which is not possible	B1	3.5b
		(1)	
			(15 marks)

10.

	Answer	Marks	AO	Guidance
(a)	$(\ln(1+x))^2 = \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right)^2$	M1	3.1a	Squaring the correct result for $\ln(1+x)$ with at least the first three terms.
	$= x^2 - x^3 + \ldots$	B1	1.1	First two terms correct.
	$\ldots + \frac{11}{12}x^4 + \ldots$	A1	2.2a	Third term correct allow any equivalent fraction or exact decimal equivalent. ISW and ignore higher power terms.
		[3]		
	Alternative method			
	$f(0) = 0$ and $f'(x) = \frac{2\ln(x+1)}{x+1}$	M1		For f(0) = 0 and correct first derivative (or stating that the second derivative is zero too).
	$f'(0)=0,\ f''(0) = 2,\ f'''(0) = -6$	A1		Correct values for first, second and third derivatives For reference: $f''(x) = \frac{2-2\ln(x+1)}{(x+1)^2}$ and $f'''(x) = \frac{2(-3+2\ln(x+1))}{(x+1)^3}$ and $f^{(4)}(x) = \frac{22-12\ln(x+1)}{(x+1)^4}$ and $f^{(4)}(0) = 22$.
	$(\ln(1+x))^2 = x^2 - x^3 + \frac{11}{12}x^4 + \ldots$	A1		All terms correct (allow simplified or exact decimal equivalents for coefficients). ISW and ignore higher power terms.
		[3]		
(b)	**DR**			
	$x^2 - x^3 + \frac{11}{12}x^4 + \ldots = 2x^3$	M1*	3.1a	Setting their expansion from part **(a)** equal to $2x^3$. Their expansion must have at least three terms with non-zero quadratic, cubic and quartic terms.
	$x^2\left(1 - 3x + \frac{11}{12}x^2\right) = 0$	M1dep*	1.1	Rearranging and factorising (allow sign slips only). Ignore higher order terms if the factor of x^2 is seen – their expansion must have contained no constant or linear terms. Stating their quadratic $1 - 3x + \frac{11}{12}x^2 = 0$ is fine for this mark. Stating either non-zero root of the correct (or their) quadratic can imply this mark.
	$x = 0.37(6690\ldots)$ or $2.89(6036\ldots)$ but $x = 0.38$ as expansion is only valid for $-1 < x \leqslant 1$.	A1	3.2a	Selects the smallest non-zero root (so must see both correct roots either exact $\frac{18 \pm 8\sqrt{3}}{11}$ or to at least **2** decimal places) and justifies with interval for validity being $-1 < x \leqslant 1$. Allow awrt 0.38 or $\frac{18-8\sqrt{3}}{11}$. Condone any indication that expansion is only valid between values of -1 and 1 or for values less than, or less than or equal, to 1. Note that including x^5 terms in expansion gives $x = 0.35997$ and including x^5 and x^6 terms in expansion give $x = 0.36504$. Ignore any attempt to work out corresponding y-coordinate.
		[3]		

11.

4(a)	$z^n + \frac{1}{z^n} \equiv e^{in\theta} + \frac{1}{e^{in\theta}} \equiv e^{in\theta} + e^{-in\theta}$	M1	1.1b
	$\equiv \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta \equiv 2\cos n\theta$ *	A1*	2.1
		(2)	
(b)	$\left(z+z^{-1}\right)^5 = 32\cos^5\theta$	B1	2.2a
	$\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1 A1	1.1b 1.1b
	$32\cos^5\theta = \left(z^5+z^{-5}\right) + 5\left(z^3+z^{-3}\right) + 10\left(z+z^{-1}\right)$ $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1	2.1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *	A1*	1.1b
		(5)	
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta \Rightarrow 16\cos^5\theta = -2\cos\theta$	B1	3.1a
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0 \Rightarrow \theta = \ldots$	M1	1.1b
	$8\cos^4\theta + 1 = 0$ has no solution so $\cos\theta = 0$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1	2.2a
		(3)	
			(10 marks)

12.

(a)	$\mathbf{r}=\begin{pmatrix}0\\11\\7\end{pmatrix}+\ldots$ or $\mathbf{r}=\begin{pmatrix}18\\2\\16\end{pmatrix}+\ldots$ or $\mathbf{r}=\ldots+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$ or $\ldots\begin{pmatrix}0\\11\\7\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$	**B1***	3.3	For two correct components – the components are • $\mathbf{r}=\ldots$ or $\vec{r}=\ldots$ ONLY • position vector, • direction vector in lowest terms with parameter.
	$\mathbf{r}=\begin{pmatrix}0\\11\\7\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$	**B1dep***	3.3	For a correct equation with direction vector in lowest terms. Condone lack of (or incorrect) range of values for parameter. Other common answers are: $\mathbf{r}=\begin{pmatrix}0\\11\\7\end{pmatrix}+\mu\begin{pmatrix}-2\\1\\-1\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}18\\2\\16\end{pmatrix}+\mu\begin{pmatrix}-2\\1\\-1\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}18\\2\\16\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$, There are other possible answers, e.g. $\mathbf{r}=\begin{pmatrix}6\\8\\10\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$,
		[2]		
(b)	$\begin{pmatrix}6\\7\\1\end{pmatrix}\times\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}8\\-4\\-20\end{pmatrix}$ $(D=)\dfrac{\left\|\left(\begin{pmatrix}0\\0\\18\end{pmatrix}-\begin{pmatrix}0\\11\\7\end{pmatrix}\right)\bullet\begin{pmatrix}8\\-4\\-20\end{pmatrix}\right\|}{\left\|\begin{pmatrix}8\\-4\\-20\end{pmatrix}\right\|}$	**M1**	3.4	Calculate vector product **n** with $\begin{pmatrix}6\\7\\1\end{pmatrix}$ and the direction vector of their T from part **(a)** with at least one component correct (following through their vectors), **and** setting up the equation $D=\dfrac{\left\|\left(\begin{pmatrix}0\\0\\18\end{pmatrix}-\mathbf{a}\right)\bullet\mathbf{n}\right\|}{\|\mathbf{n}\|}$ where **a** is a point on their T from part **(a)** and their vector **n**. Condone lack of modulus signs in equation. Note that $\pm$ these vectors are valid as are scalar multiples of **n**.
	$=\dfrac{22\sqrt{30}}{15}(=8.033264\ldots)>5$ so yes (the site) passes Test 1.	**A1**	2.2a	Correct calculation (either exact e.g. $\dfrac{176}{4\sqrt{30}}$) or at least **2** significant figures), including reference to 5 and conclusion.
		[2]		

(c)	$(\overrightarrow{PQ}=)\pm\left(\begin{pmatrix}0\\0\\18\end{pmatrix}+\lambda\begin{pmatrix}6\\7\\1\end{pmatrix}-\begin{pmatrix}0\\11\\7\end{pmatrix}\right)\quad\left(=\pm\begin{pmatrix}6\lambda\\7\lambda-11\\\lambda+11\end{pmatrix}\right)$	B1	3.4	**No MR in this part.** For $(0,11,7)$ to a general point on *S*. Allow correct un-simplified. Can be implied by a correct expression for the distance (or distance squared)
	$\left(\lvert\overrightarrow{PQ}\rvert=\right)\sqrt{(6\lambda)^2+(7\lambda-11)^2+(\lambda+11)^2}$	B1	3.4	Expression for $\lvert\overrightarrow{PQ}\rvert$ or $\lvert\overrightarrow{PQ}\rvert^2$ - allow un-simplified.
	$(6\lambda)^2+(7\lambda-11)^2+(\lambda+11)^2\geqslant 19^2\quad(86\lambda^2-132\lambda-119\geqslant 0)$ $\Rightarrow\lambda\geqslant 2.17...$ or $\lambda\leqslant -0.637...$ but $0\leqslant\lambda\leqslant 2$ so no, it's not possible.	B1	3.1b	Solving inequality/equation for λ and concluding no + reason. Condone "no" as conclusion. Critical values of λ correct to at least **2** sf. Allow strict inequalities.
		[3]		
	Alternative method			
	Distance between $(0,11,7)$ and $(0,0,18)$ is $\sqrt{11^2+(7-18)^2}$	B1		Find distance (or distance squared) between (0, 11, 7) and (0, 0, 18) – allow un-simplified.
	Distance between $(0,11,7)$ and $(12,14,20)$ is $\sqrt{12^2+(14-11)^2+(20-7)^2}$	B1		Find distance (or distance squared) between (0, 11, 7) and (12, 14, 20) – allow un-simplified
	$11\sqrt{2}=15.5563...$ and $\sqrt{322}=17.9443...$ which are both less than 19 and the two point (0, 0, 18) and (12, 14, 20) are the points furthest from (0, 11, 7) so no, it's not possible.	B1		Correct values given to at least **3** sf (or in a form in which all three can be compared e.g. $\sqrt{322},\sqrt{242}$ and $\sqrt{361}$) and some indication that these two points are the furthest from the camera and conclude no.