

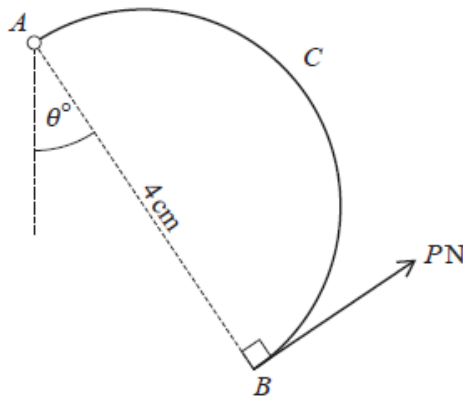
Topic X4 Centre of mass (Post-TT A) [42]

1.

The region bounded by the x -axis, the y -axis, the line $x = \ln 3$, and the curve $y = e^{-x}$ for $0 \leq x \leq \ln 3$, is occupied by a uniform lamina. Find, in an exact form, the coordinates of the centre of mass of this lamina. [9]

(Total 9 marks)

2.



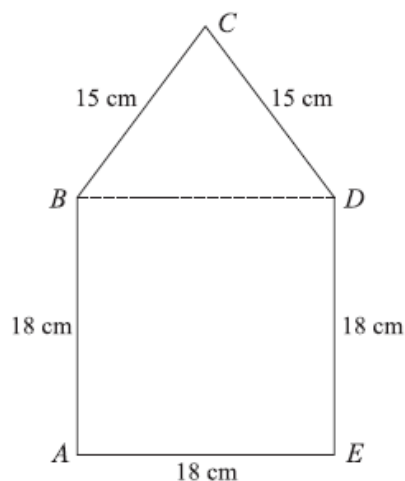
A uniform semicircular arc ACB is freely pivoted at A . The arc has mass 0.3 kg and is held in equilibrium by a force of magnitude PN applied at B . The line of action of this force lies in the same plane as the arc, and is perpendicular to AB . The diameter AB has length 4 cm and makes an angle of θ° with the downward vertical (see diagram).

(i) Given that $\theta = 0$, find the magnitude of the force acting on the arc at A . [6]

(ii) Given instead that $\theta = 30$, find the value of P . [4]

(Total 10 marks)

3.



A uniform lamina $ABCDE$ consists of a square and an isosceles triangle. The square has sides of 18 cm and $BC = CD = 15 \text{ cm}$ (see diagram).

(i) Taking x - and y -axes along AE and AB respectively, find the coordinates of the centre of mass of the lamina. [7]

(ii) The lamina is freely suspended from B . Calculate the angle that BD makes with the vertical. [2]

(Total 9 marks)

4.

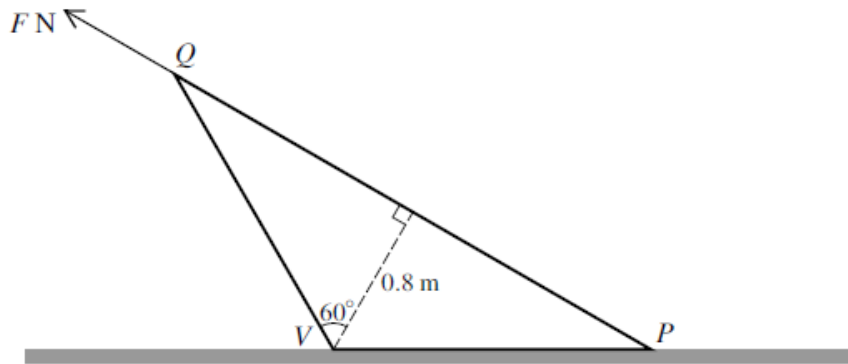


Fig. 1

A uniform solid cone of height 0.8 m and semi-vertical angle 60° lies with its curved surface on a horizontal plane. The point P on the circumference of the base is in contact with the plane. V is the vertex of the cone and PQ is a diameter of its base. The weight of the cone is 550 N. A force of magnitude F N and line of action PQ is applied to the base of the cone (see Fig. 1). The cone topples about V without sliding.

- (i) Calculate the least possible value of F . [4]

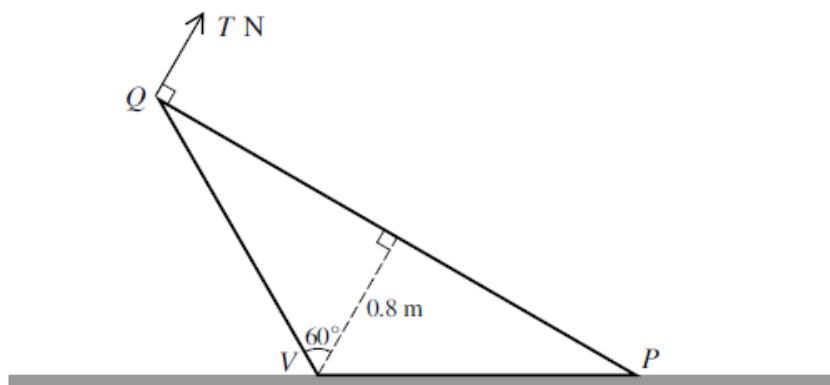


Fig. 2

The force of magnitude F N is removed and an increasing force of magnitude T N acting upwards in the vertical plane of symmetry of the cone and perpendicular to PQ is applied to the cone at Q (see Fig. 2). The coefficient of friction between the cone and the horizontal plane is μ .

- (ii) Given that the cone slides before it topples about P , calculate the greatest possible value for μ .

[10]

(Total 14 marks)