

## Topic X4 Centre of mass (Post-TT A) [42] MARKSCHEME

1.

Area is $\int_0^{\ln 3} e^{-x} dx$ $= \left[ -e^{-x} \right]_0^{\ln 3} \quad \left( = \frac{2}{3} \right)$	M1	<i>Limits not required</i>
	A1	For $-e^{-x}$
$\int xy dx = \int_0^{\ln 3} xe^{-x} dx$ $= \left[ -xe^{-x} - e^{-x} \right]_0^{\ln 3} \quad \left( = \frac{2}{3} - \frac{1}{3} \ln 3 \right)$	M1	<i>Limits not required</i>
	M1	Integration by parts
	A1	For $-xe^{-x} - e^{-x}$
$\bar{x} = \frac{\frac{2}{3} - \frac{1}{3} \ln 3}{\frac{2}{3}} = 1 - \frac{1}{2} \ln 3$	A1	
$\int \frac{1}{2} y^2 dx = \int_0^{\ln 3} \frac{1}{2} (e^{-x})^2 dx$ $= \left[ -\frac{1}{4} e^{-2x} \right]_0^{\ln 3} \quad \left( = \frac{2}{9} \right)$	M1	$\int (e^{-x})^2 dx$ or $\int (-\ln y)y dy + (\frac{1}{3} \ln 3) \times \frac{1}{6}$
	A1	$-\frac{1}{4} e^{-2x}$ or $-\frac{1}{2} y^2 \ln y + \frac{1}{4} y^2$ (dep on M1)
$\bar{y} = \frac{\frac{2}{9}}{\frac{2}{3}} = \frac{1}{3}$	A1	<i>Max penalty of 1 mark for correct answers in an unacceptable form (eg decimals)</i>
<b>[9]</b>		

2.

(i)	$x_G = (2 \times 2) / \pi$  $P(\text{or } X) \times 4 = 0.3g \times x_G$ $Y = 0.3g$ Use $R^2 = X^2 + Y^2$ to find $R$ $R = 3.09 \text{ N}$	B1 *M1 A1ft B1 dep*M1 A1 [6]	$x_G = 1.2732\dots$ May be seen in (ii), mark only once. Take moments about $A$ or $B$ $P = 0.9358\dots$ ft their $x_G$ for this mark.
(ii)	$P \times 4 =$ $0.3g \times (2 \sin 30 + x_G \sin 60)$ $P = 1.55$	M1 A1 A1 A1 [4]	Attempt at moments, force $\times$ distance = $0.3g \times$ distance  $0.3g \times 2.1026\dots$ $1.545453\dots$

3.

(i)	$\bar{x} = 9$ c of m of $\Delta$ 4 cm above BD  $(324 + 108) \text{ (m)} \bar{y} =$ $324 \text{ (m)} \times 9 + 108 \text{ (m)} \times (18+4)$ $432 \bar{y}$ $324 \times 9 \quad (18^2 \times 9)$ $108 \times (18 + 4)$ $\bar{y} = 12.25$	B1 B1  M1 A1 A1 A1 A1	7	ignore any working  <b>8 cm below C/see their diagram</b> $432 \bar{y} = 108 \times 8 + 18^2 (12 + 9)$ from C left hand side 1 <sup>st</sup> term on right hand side 2916 2 <sup>nd</sup> term on right hand side 2376 $5292 = 432 \bar{y}$ or $49/4$
(ii)	$\tan \theta = 5.75/9$ $\theta = 32.6^\circ$ or $147.4^\circ$	M1 A1✓	2	must be $\dots/9$ $\sqrt{\tan^{-1} ((18 - \text{their } \bar{y})/9)}$ or $180^\circ \dots$
				<b>9</b>

4.

<p>7 i</p>	<p><math>F \times 0.8 =</math>  <math>0.6\cos 60 \times 550</math>  <math>F = 206.25</math></p>	<p>M1  A1  A1  A1  [4]</p>	<p>Attempt at moments   Accept 206, cao</p>
<p>ii</p>	<p><math>T \times 2 \times 0.8/\tan 30</math>  <math>=</math>  <math>550 \times (0.8/\sin 30 - 0.6\cos 60)</math>  <math>T = 258</math>   <math>R = 550 - T\cos 30</math>  <math>Fr = T\sin 30</math>  <math>\mu = 129/326.6</math>  <math>\mu = 0.395</math></p>	<p>M1*  A1  M1*  A1  A1  M1*  A1  B1*  M1dep*  A1  [10]</p>	<p>Moment of T about P  <math>T \times 2.77</math>  Moment of weight about P  <math>550 \times (1.6 - 0.3)</math>  Accept to 2sf  Resolving vertically, 3 terms needed  Value for T not required  Value for T not required; accept <math>&lt;</math> or <math>\leq</math>  For correct use of <math>F = \mu R</math>, <math>R \neq 550</math></p>
<p>OR</p>	<p><math>T \times 0.8/\tan 30 + 550 \times 0.6\cos 60 = R \times 0.8/\cos 60</math>   <math>R = 550 - T\cos 30</math>  Solve for T or R  <math>T = 258</math> or <math>R = 326.5625</math>  <math>Fr = T\sin 30</math>  <math>\mu = 129/326.6</math>  <math>\mu = 0.395</math></p>	<p>M1*  A2  M1*  A1  M1  A1  A1  B1*  M1dep*  A1  [10]</p>	<p>Moments about V, 3 terms needed  A1 for two terms correct  Resolving vertically, 3 terms needed   Accept to 2sf  Value for T not required; accept <math>&lt;</math> or <math>\leq</math>  For correct use of <math>F = \mu R</math>, <math>R \neq 550</math></p>
<p>OR</p>	<p><math>Fr \times 1.6\cos 30 + 550 \times (1.6\sin 30 + 0.6\sin 30) =</math>  <math>R \times (1.6 + 1.6\sin 30)</math>   <math>R = 550 - T\cos 30</math>  <math>Fr = T\sin 30</math>  Solving for at least one of R, Fr, or T  Either <math>R = 326.5625</math>, or <math>Fr = 129(.0017008)</math>, or <math>T = 258</math>  <math>\mu = 129/326.6</math>  <math>\mu = 0.395</math></p>	<p>M1*   A2  M1*  A1  B1*  M1  A1  M1dep*  A1  [10]</p>	<p>Moments about Q, 3 terms needed   A1 for two terms correct  Resolving vertically, 3 terms needed   accept <math>&lt;</math> or <math>\leq</math>   Only one needed. Accept to 2sf.  For correct use of <math>F = \mu R</math>, <math>R \neq 550</math></p>