

Topic X4 Centre of mass (Pre-TT A) [35]

1.

The region R is bounded by the curve $y = \sqrt{4a^2 - x^2}$ for $0 \leq x \leq a$, the x -axis, the y -axis and the line $x = a$, where a is a positive constant. The region R is rotated through 2π radians about the x -axis to form a uniform solid of revolution. Find the x -coordinate of the centre of mass of this solid. [7]

2.

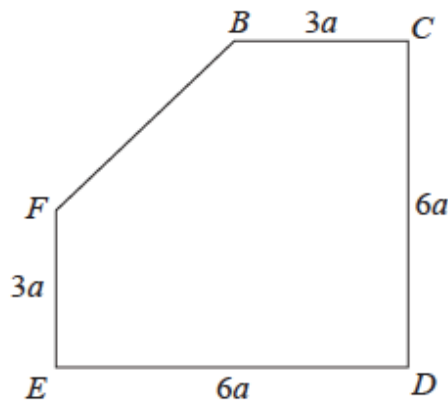


Figure 2

The uniform lamina, L , with $BC = EF = 3a$ and $CD = DE = 6a$, shown in Figure 2, is formed by removing an isosceles triangle, whose equal sides are of length $3a$, from a square of side $6a$.

(a) Show that the centre of mass of L is $\frac{23}{7}a$ from EF .

(4)

A uniform triangular lamina, ABF , made from a different material to L , is fixed to L to form the square template, $ABCDEF$, shown in Figure 3.

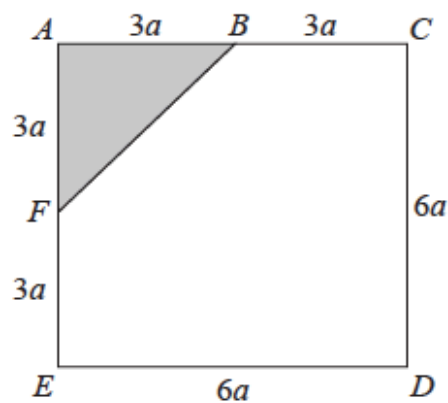


Figure 3

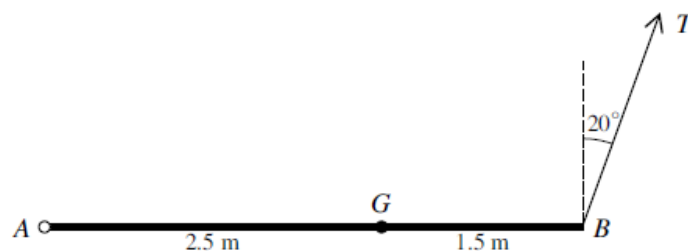
The lamina L has mass M per unit area. The triangular lamina ABF has mass $3M$ per unit area.

The template is freely suspended from C and hangs in equilibrium, with CD at an angle θ° to the downward vertical.

(b) Find the value of θ .

(6)

3.



A non-uniform beam AB of length 4 m and mass 5 kg has its centre of mass at the point G of the beam where $AG = 2.5$ m. The beam is freely suspended from its end A and is held in a horizontal position by means of a wire attached to the end B . The wire makes an angle of 20° with the vertical and the tension is T N (see diagram).

(i) Calculate T . [3]

(ii) Calculate the magnitude and the direction of the force acting on the beam at A . [7]

4.

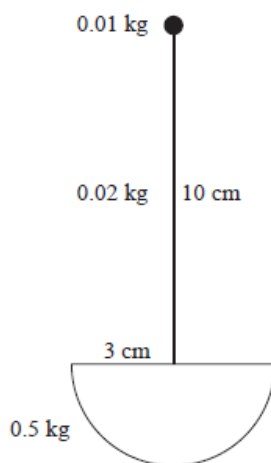


Fig. 1

A toy is constructed by attaching a small ball of mass 0.01 kg to one end of a uniform rod of length 10 cm whose other end is attached to the centre of the plane face of a uniform solid hemisphere with radius 3 cm. The rod has mass 0.02 kg, the hemisphere has mass 0.5 kg and the rod is perpendicular to the plane face of the hemisphere (see Fig. 1).

(i) Show that the distance from the ball to the centre of mass of the toy is 10.7 cm, correct to 1 decimal place. [4]

(ii)

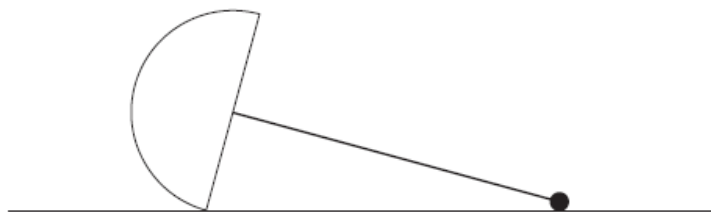


Fig. 2

The toy lies on horizontal ground in a position such that the ball is touching the ground (see Fig. 2). Determine whether the toy is lying in equilibrium or whether it will move to a position where the rod is vertical. [4]