Topic X4 Centre of mass (Pre-TT A) [35] MARKSCHEME

1.

Volume is $\int_0^a \pi(4a^2 - x^2)\,dx = \pi\left[4a^2x - \frac{1}{3}x^3\right]_0^a$	M1	π *may be omitted throughout* (Limits not required)
$= \frac{11}{3}\pi a^3$	A1	
$\int_0^a \pi x(4a^2 - x^2)\,dx$	M1	
$= \pi\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^a$	A1	(Limits not required)
$= \frac{7}{4}\pi a^4$	A1	
$\bar{x} = \frac{\frac{7}{4}\pi a^4}{\frac{11}{3}\pi a^3}$	M1	for $\frac{\int x y^2\,dx}{\int y^2\,dx}$
$= \frac{21}{44}a$	A1	
	7	

2.

Question	Scheme	Marks	AOs
2(a)	Area of $L = 36a^2 - \frac{9}{2}a^2 \left(= \frac{63}{2}a^2\right)$	B1	1.2
	Moments equation to find the distance	M1	2.1
	$36a^2 \times 3a - \frac{9}{2}a^2 \times a \left(= \frac{207}{2}a^3\right) = \frac{63}{2}a^2 \times \bar{x}$	A1ft	1.1b
	$\Rightarrow \bar{x} = \frac{207}{63}a = \frac{23}{7}a$ *	A1*	2.2a
		(4)	
(b)	Mass ratios $63 : 27 : 90$	B1	1.1b
	Complete strategy to find the centre of mass	M1	3.1b
	$\frac{23}{7}a \times \frac{63}{2}a^2M + a \times \frac{9}{2}a^2 \times 3M = d \times a^2M\left(\frac{63}{2} + 3 \times \frac{9}{2}\right)$ $\left(117a^3 = d \times 45a^2\right)$	A1ft	1.1b
	$d = \frac{13}{5}a$	A1	1.1b
	$\tan\theta = \frac{6a - d}{d}\left(= \frac{17}{13}\right)$	M1	3.1b
	$\theta = 52.6$ (53 or better)	A1	1.1b
		(6)	
			(10 marks)

Notes:		
(a)	B1	Correct area of *L* seen or implied
	M1	moments about *EF* or a parallel axis. Condone slips but needs to be dimensionally correct and a clear attempt to combine elements correctly.
	A1ft	Correct unsimplified moments equation. Follow their $\frac{63}{2}a^2$
	A1*	Obtain given result from correct working
(b)	B1	Correct mass ratios – any equivalent form
	M1	Complete strategy: use moments to find distance of centre of mass of template from any side
	A1ft	Correct unsimplified equation. Follow their mass ratios
	A1	Correct distance: $d = \frac{13}{5}a$ from *AE* or *AC*, $d = \frac{17}{5}a$ from *CD* or *DE*
	M1	Complete strategy: use of their distances to find a relevant angle. For their *d*. Condone reciprocal
	A1	2 s.f. or better 52.59464....

3.

(i)	4Tcos20° = 5 x g x 2.5 T = 32.6 N	M1 A1 A1 **3**	Using moments; allow sin/cos mix Allow with omission of g
(ii)	X = Tsin20° X = 11.1 FT Y + Tcos20° = 5 x g or 2.5Y = 1.5 x Tcos20 or 4Y = 1.5 x 5g Y = 18.4 FT R = √(X² + Y²) or tan⁻¹ (Y/X) or tan⁻¹ (X/Y) R = 21.5 N θ = 58.8° above the horizontal	M1 A1 M1 A1 M1 A1 A1 **7**	allow sin/cos mix FT their T FT their T, but not from omission of g X ≠ 0, Y ≠ 0 or 31.2° to left of vertical **10**

4.

(i)	3/8 x 3 (1.125) 0.53d = 5x0.02 + (10 + 3/8x3) x 0.5 d = 10.7	B1 M1 A1 A1 **4**	c.o.m. hemisphere 0.53e=3x5/8x0.5+8x0.02+13x .01 0.53f=3x3/8x0.5-5x0.02-10x0.01 **AG** (e = 2.316 f = 0.684)
ii)	Attempt to calc a pair relevant to P,G OP=0.9 (pair) , p= 73.3° q=16.7° r=76.9° (77.2°) , s=13.1°(12.8°) AC=0.86 , BC=0.67 , AD=10.4 BD=10.2 r > p , s < q , p + s < 90 , 0.67 < 0.86 , 10.2 < 10.4 it is in equilibrium	M1 A1 M1 A1 **4**	distance / angle not a complimentary pair make relevant comparison 0.7 < 0.9 (OG < OP) 10.7 < 10.9 **8**