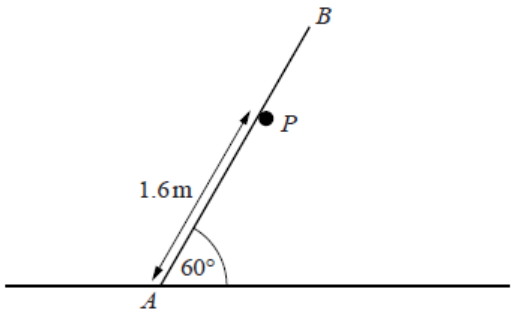
Topic X4 Centre of mass (Pre-TT B) [45]

1.



A uniform rod AB of mass 10 kg and length 2.4 m rests with A on rough horizontal ground. The rod makes an angle of 60° with the horizontal and is supported by a fixed smooth peg P. The distance AP is 1.6 m (see diagram).

(i) Calculate the magnitude of the force exerted by the peg on the rod. **[3]**

(ii) Find the least value of the coefficient of friction between the rod and the ground needed to maintain equilibrium. **[5]**

2.

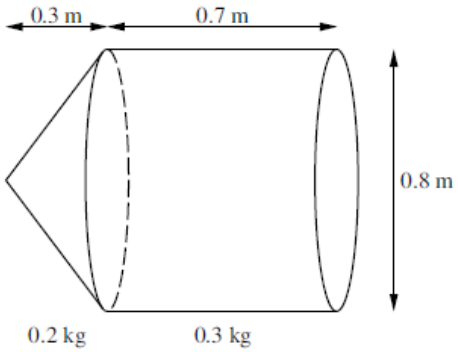


Fig. 1

A uniform conical shell has mass 0.2 kg, height 0.3 m and base diameter 0.8 m. A uniform hollow cylinder has mass 0.3 kg, length 0.7 m and diameter 0.8 m. The conical shell is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

(i) Show that the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m. **[4]**

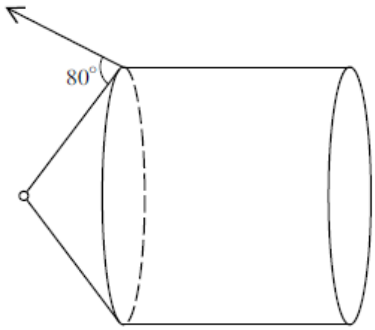


Fig. 2

The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of 80° with the slant edge of the conical shell (see Fig. 2).

(ii) Calculate the tension in the wire. **[4]**

3.

The region bounded by the curve $y = 2x + x^2$ for $0 \leqslant x \leqslant 3$, the x-axis, and the line $x = 3$, is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [9]

(Total 9 marks)

4.

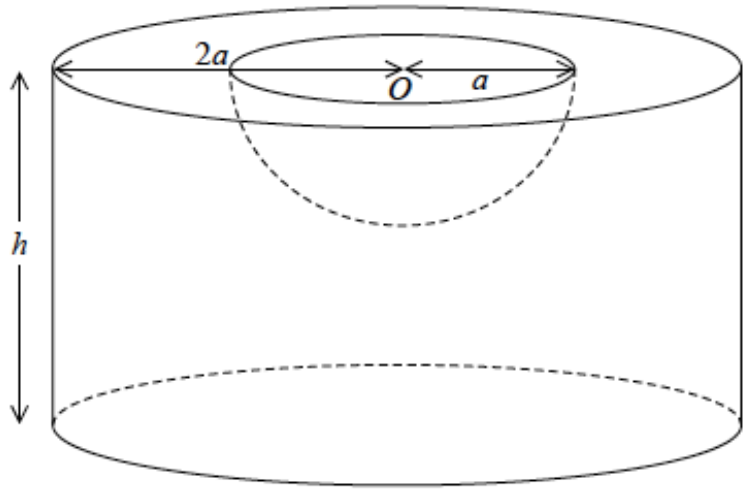


Figure 2

A uniform solid cylinder has radius $2a$ and height h ($h > a$).

A solid hemisphere of radius a is removed from the cylinder to form the vessel V.

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The centre O of the hemisphere is also the centre of the upper plane face of the cylinder, as shown in Figure 2.

(a) Show that the centre of mass of V is $\dfrac{3(8h^2 - a^2)}{8(6h - a)}$ from O.

(5)

The vessel V is placed on a rough plane which is inclined at an angle ϕ to the horizontal.

The lower plane circular face of V is in contact with the inclined plane.

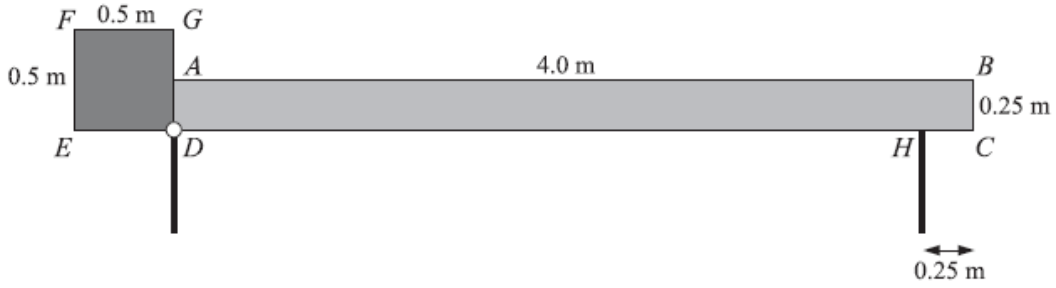
Given that $h = 5a$, the plane is sufficiently rough to prevent V from slipping and V is on the point of toppling,

(b) find, to three significant figures, the size of the angle ϕ.

(4)

(Total 9 marks)

5.



A barrier is modelled as a uniform rectangular plank of wood, $ABCD$, rigidly joined to a uniform square metal plate, $DEFG$. The plank of wood has mass 50 kg and dimensions 4.0 m by 0.25 m. The metal plate has mass 80 kg and side 0.5 m. The plank and plate are joined in such a way that CDE is a straight line (see diagram). The barrier is smoothly pivoted at the point D. In the closed position, the barrier rests on a thin post at H. The distance CH is 0.25 m.

(i) Calculate the contact force at H when the barrier is in the closed position. [3]

In the open position, the centre of mass of the barrier is vertically above D.

(ii) Calculate the angle between AB and the horizontal when the barrier is in the open position. [8]

(Total 11 marks)