Topic X5 Variable forces and oblique collisions (Post-TT A) [50]

1.

A bungee jumper of mass 70 kg is joined to a fixed point O by a light elastic rope of natural length 30 m and modulus of elasticity 1470 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Find the distance fallen by the jumper when maximum speed is reached. [4]

(ii) Show that this maximum speed is 26.9 m s^{-1}, correct to 3 significant figures. [4]

(iii) Find the extension of the rope when the jumper is at the lowest position. [4]

(Total 12 marks)

2.

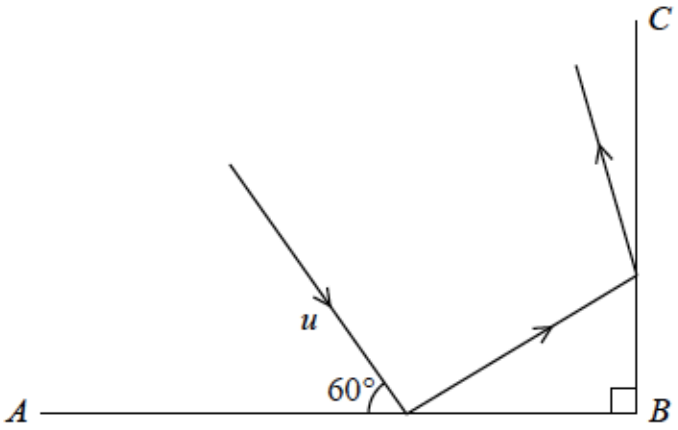


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards AB with speed u m s^{-1} on a path at an angle of 60° to AB. The ball hits AB and then hits BC.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall AB is $\frac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall BC is $\sqrt{\frac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

(Total 9 marks)

3.

[In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

(a) Find the velocity of B immediately after the collision.

(7)

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision.

(2)

(Total 9 marks)

4.

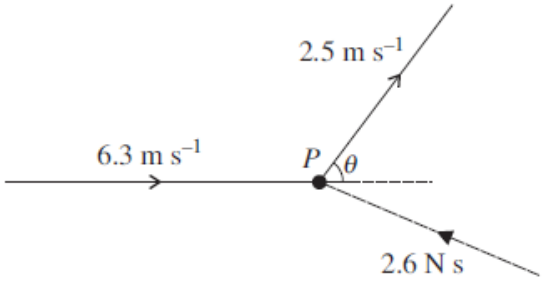
A stone of mass 0.125 kg falls freely under gravity, from rest, until it has travelled a distance of 10 m. The stone then continues to fall in a medium which exerts an upward resisting force of $0.025v$ N, where $v\,\text{m s}^{-1}$ is the speed of the stone t s after the instant that it enters the resisting medium.

(i) Show by integration that $v = 49 - 35e^{-0.2t}$. **[8]**

(ii) Find how far the stone travels during the first 3 seconds in the medium. **[4]**

(Total 12 marks)

5.



A particle P of mass 0.5 kg is moving in a straight line with speed $6.3\,\text{m s}^{-1}$. An impulse of magnitude 2.6 N s applied to P deflects its direction of motion through an angle θ, and reduces its speed to $2.5\,\text{m s}^{-1}$ (see diagram). By considering an impulse-momentum triangle, or otherwise,

(i) show that $\cos\theta = 0.6$, **[4]**

(ii) find the angle that the impulse makes with the original direction of motion of P. **[4]**

(Total 8 marks)