## Topic X5 Variable forces and oblique collisions (Post-TT) [50] MARKSCHEME

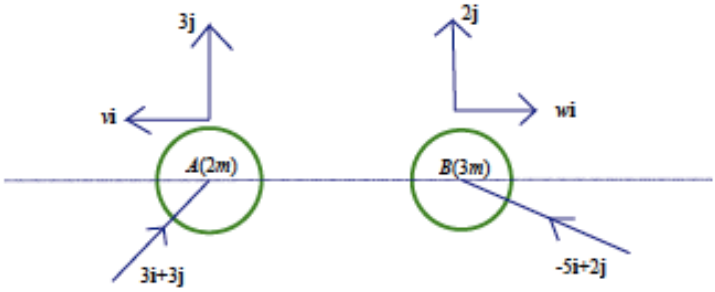
1.

| | | | | |
|---|---|---|---|---|
| **(i)** | $T = 1470x/30$ | B1 | | |
| | $[49x = 70x9.8]$ | M1 | | For using T = mg |
| | $x = 14$ | A1 | | |
| | Distance fallen is 44m | A1ft | 4 | |
| **(ii)** | PE loss = $70g(30 + 14)$ | B1ft | | |
| | EE gain = $1470x14^2/(2x30)$ | B1ft | | |
| | $[½\ 70v^2 = 30184 – 4802]$ | M1 | | For a linear equation with terms representing KE, PE and EE changes. |
| | Speed is $26.9ms^{-1}$ | A1 | 4 | AG |
| **OR** | | | | |
| **(ii)** | $[0.5\ v^2 = 14g – 68.6 + 30g]$ | M1 | | For using Newton's 2nd law ($vdv/dx = g – 0.7x$), integrating ($0.5\ v^2 = gx – 0.35x^2 + k$), using $v(0)^2 = 60g$ → $k = 30g$, and substituting $x = 14$. |
| | For $14g + 30g$ | B1ft | | |
| | For $\mp 68.6$ | B1ft | | Accept in unsimplified form. |
| | Speed is $26.9ms^{-1}$ | A1 | 4 | AG |
| **(iii)** | PE loss = $70g(30 + x)$ | B1ft | | |
| | EE gain = $1470x^2/(2x30)$ | B1ft | | |
| | $[x^2 – 28x – 840 = 0]$ | M1 | | For using PE loss = KE gain to obtain a 3 term quadratic equation. |
| | Extension is 46.2m | A1 | 4 | |
| **OR** | | | | |
| **(iii)** | | M1 | | For identifying SHM with $n^2 = 1470/(70x30)$ |
| | | M1 | | For using $v_{max} = An$ |
| | $A = 26.9 / \sqrt{0.7}$ | A1 | | |
| | Extension is 46.2m | A1 | 4 | |

2.

| | | | |
|---|---|---|---|
| **(a)** | Complete strategy to find the kinetic energy after the second impact | M1 | |
| | Parallel to $AB$ after collision: $u\cos 60°$ | M1 | |
| | Perpendicular to $AB$ after collision: $\frac{1}{\sqrt{3}}u\sin 60°$ | M1 | |
| | Components of velocity after first impact: $\frac{u}{2}$, $\frac{u}{2}$ | A1 | |
| | Parallel to $BC$ after collision: $\frac{u}{2}$ $\left(u\times\frac{1}{\sqrt{3}}\sin 60°\right)$ | M1 | |
| | Perpendicular to $BC$ after collision: $\sqrt{\frac{2}{5}}\times\frac{u}{2}\left(=\frac{1}{\sqrt{10}}u\right)$ $\left(\sqrt{\frac{2}{5}}\times u\cos 60°\right)$ | M1 | |
| | Components of velocity after second impact: $\frac{u}{2}$, $\frac{u}{\sqrt{10}}$ | A1 | |
| | Final KE $=\frac{1}{2}m\left(\frac{u^2}{4}+\frac{u^2}{10}\right)$ $\left(=\frac{mu^2}{2}\times\frac{7}{20}\right)$ | | |
| | Fraction of initial KE $=\frac{\frac{mu^2}{2}\times\frac{7}{20}}{\frac{mu^2}{2}}=\frac{7}{20}=35\%$ * | A1* | 2.2a |
| | | **(8)** | |
| **(b)** | The answer is too large - rough surface means resistance so final speed will be lower | B1 | 3.5a |
| | | **(1)** | |

3.

| | | |
|---|---|---|
| **(a)** |  | |
| | Overall strategy to find $\mathbf{V}_A$ | M1 |
| | Velocity of $A$ perpendicular to loc after collision $= 3\mathbf{j}$ (m s$^{-1}$) | B1 |
| | CLM parallel to loc | M1 |
| | $2m \times 3 - 3m \times 5 = 3mw - 2mv \quad (-9 = 3w\text{-}2v)$ | A1 |
| | Correct use of impact law | M1 |
| | $v + w = \frac{1}{4}(3+5) \ (= 2)$ | A1 |
| | Solve for $w$ $\quad 3w - 2v = -9$, $\quad 2v + 2w = 4$ | |
| | $\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ (m s$^{-1}$), | A1ft |
| | | **(7)** |
| **(b)** | $\cos\theta = \frac{(-5\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+2\mathbf{j})}{\sqrt{29}\sqrt{5}}$ | M1 |
| | $\theta = 41.63...° = 42°$ (nearest degree) | A1 |
| | Alternative method: $\tan^{-1} 2 - \tan^{-1} \frac{2}{5} = 41.63...° = 42°$ (nearest degree) | |
| | | **(2)** |

4.

| | | | |
|---|---|---|---|
| **(i)** | Initial speed in medium is $\sqrt{2g \times 10}$ $(= 14)$ | B1 | |
| | $[0.125dv/dt = 0.125g - 0.025v]$ | M1 | For using Newton's second law with a = dv/dt (3 terms required) |
| | $\int \frac{5dv}{5g - v} = \int dt$ | M1 | For separating variables and attempt to integrate |
| | $-5\ln(5g - v) = t\ (+A)$ | A1 | |
| | $[-5\ln 35 = A]$ | M1 | For using v(0) = 14 |
| | $t = 5\ln\{35/(49 - v)\}$ | A1 | |
| | | M1 | For method of transposition |
| | $v = 49 - 35e^{-0.2t}$ | A1 | AG |
| | | [8] | |
| **(ii)** | | M1 | For integrating to find x(t) |
| | $x = 49t + 175e^{-0.2t}\ (+B)$ | A1 | |
| | $[x(3) = (49x3 + 175e^{-0.6}) - (0 + 175)]$ | M1 | For using limits 0 to 3 or for using x(0) = 0 and evaluating x(3) |
| | Distance is 68.0m | A1 | |
| | | [4] | |

5.

| | | | |
|---|---|---|---|
| **(i)** | For triangle sketched with sides (0.5)2.5 and (0.5)6.3 and angle $\theta$ correctly marked OR Changes of velocity in i and j directions $2.5\cos\theta - 6.3$ and $2.5\sin\theta$, respectively. | B1 | May be implied in subsequent working. |
| | For sides 0.5x2.5, 0.5x6.3 and 2.6 (or 2.5, 6.3 and 5.2) OR $-2.6\cos\alpha = 0.5(2.5\cos\theta - 6.3)$ and $2.6\sin\alpha = 0.5(2.5\sin\theta)$ | B1ft | May be implied in subsequent working. |
| | $[5.2^2 = 2.5^2 + 6.3^2 - 2x2.5x6.3\cos\theta$ OR $2.6^2 = 0.5^2\{(2.5\cos\theta - 6.3)^2 + (2.5\sin\theta)^2]$ | M1 | For using cosine rule in triangle or eliminating $\alpha$. |
| | $\cos\theta = 0.6$ | A1 | AG |
| | | [4] | |
| **(ii)** | | M1 | For appropriate use of the sine rule or substituting for $\theta$ in one of the above equations in $\theta$ and $\alpha$ |
| | $\sin\alpha = 2.5x0.8/5.2$ OR $-2.6\cos\alpha = 0.5(2.5x0.6 - 6.3)$ | A1 | |
| | | M1 | For evaluating $(180 - \alpha)^\circ$ or $(\pi - \alpha)^c$ |
| | Impulse makes angle of 157° or $2.75^c$ with original direction of motion of P. | A1 | |
| | | [4] | |
| | | | SR (relating to previous 2 marks; max 1 mark out of 2) |
| | | | $\alpha = 23^\circ$ or $0.395^c$ B1 |