Topic X5 Variable forces and oblique collisions (Post-TT B) [47]

1.

The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass m kg and is released from rest. When a grain has fallen a distance x m, it has speed $v\,\text{m s}^{-1}$. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude mkv^2 N, where k is a positive constant.

(i) Show that $\left(\frac{v}{g - kv^2}\right)\frac{\mathrm{d}v}{\mathrm{d}x} = 1$. [2]

(ii) Find v^2 in terms of k, g and x. Hence show that, as x becomes large, the limiting value of v is $\sqrt{\frac{g}{k}}$. [7]

(iii) Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find k. [3]

(Total 12 marks)

2.

A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $(2\mathbf{i} - \mathbf{j})\,\text{N s}$.

Show that the kinetic energy gained by P as a result of the impulse is 12 J.

(6)

(Total 6 marks)

3.

A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $0.75mg$ N. The other end of the string is attached to a fixed point O of a smooth plane inclined at 30° to the horizontal. P is released from rest at O and moves down the plane.

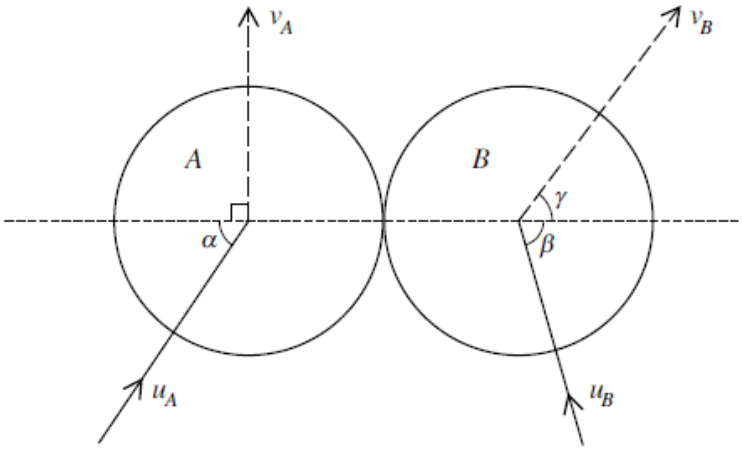
(i) Show that the maximum speed of P is reached when the extension of the string is 0.8 m. [3]

(ii) Find the maximum speed of P. [4]

(iii) Find the maximum displacement of P from O. [4]

(Total 11 marks)

4.



Two uniform smooth identical spheres A and B are moving towards each other on a horizontal surface when they collide. Immediately before the collision A and B are moving with speeds $u_A\,\text{m s}^{-1}$ and $u_B\,\text{m s}^{-1}$ respectively, at acute angles α and β, respectively, to the line of centres. Immediately after the collision A and B are moving with speeds $v_A\,\text{m s}^{-1}$ and $v_B\,\text{m s}^{-1}$ respectively, at right angles and at acute angle γ, respectively, to the line of centres (see diagram).

(i) Given that $\sin\beta = 0.96$ and $\dfrac{v_B}{u_B} = 1.2$, find the value of $\sin\gamma$. **[2]**

(ii) Given also that, before the collision, the component of A's velocity parallel to the line of centres is $2\,\text{m s}^{-1}$, find the values of u_B and v_B. **[5]**

(iii) Find the coefficient of restitution between the spheres. **[3]**

(iv) Given that the kinetic energy of A immediately before the collision is $6.5m\,\text{J}$, where $m\,\text{kg}$ is the mass of A, find the value of v_A. **[2]**

(Total 12 marks)

5.

A small ball of mass $0.8\,\text{kg}$ is moving with speed $10.5\,\text{m s}^{-1}$ when it receives an impulse of magnitude $4\,\text{N s}$. The speed of the ball immediately afterwards is $8.5\,\text{m s}^{-1}$. The angle between the directions of motion before and after the impulse acts is α. Using an impulse-momentum triangle, or otherwise, find α. **[6]**

(Total 6 marks)