Topic X5 Variable forces and oblique collisions (Post-TT B) [47] MARKSCHEME

1.

(i) $[mg - mkv^2 = ma]$	M1		For using Newton's second law
$(v\,dv/dx)/(g - kv^2) = 1$	A1	2	AG
(ii) $[-\frac{1}{2}[\ln(g - kv^2)]/k = x \quad (+C)]$	M1		For separating variables and attempting to integrate
$[-(\ln g)/2k = C]$	M1		For using v(0) = 0 to find C
$x = [-\frac{1}{2}[\ln\{(g - kv^2)/g\}]/k$	A1		Any equivalent expression for x
$[\ln\{(g - kv^2)/g\} = \ln(e^{-2kx})]$	M1		For expressing in the form $\ln f(v^2) = \ln g(x)$ or equivalent
$v^2 = (1 - e^{-2kx})g/k$	A1		
	M1		For using $e^{-Ax} \rightarrow 0$ for +ve A
Limiting value is $\sqrt{g/k}$	A1ft	7	AG
(iii) $[1 - e^{-600k} = 0.81]$	M1		For using $v^2(300) = 0.9^2 g/k$
$[-600k = \ln(0.19)]$	M1		For using logarithms to solve for k
$k = 0.00277$	A1	3	

2.

Use Impulse-momentum principle	M1
$2\mathbf{i} - \mathbf{j} = 0.5\mathbf{v} - 0.5(4\mathbf{i} + \mathbf{j})$	A1
$\frac{1}{2}\mathbf{v} = 4\mathbf{i} - \frac{1}{2}\mathbf{j}, \quad \mathbf{v} = 8\mathbf{i} - \mathbf{j}$ (m s^{-1})	A1
Use of $KE = \frac{1}{2}m\lvert\mathbf{v}\rvert^2 - \frac{1}{2}m\lvert\mathbf{u}\rvert^2$	M1
$= \frac{1}{2} \times 0.5 \times \{(64+1) - (16+1)\}$	A1
$= \frac{1}{4} \times 48 = 12$ (J) *	A1*
	(6)

3.

5	(i)	M1		For using Newton's second law with a = 0
	$mg\sin30^\circ = 0.75mgx/1.2$	A1		
	Extension is 0.8m	A1	3	AG
	(ii) PE loss = $mg(1.2 + 0.8)\sin 30^\circ$ (mg)	B1		
	EE gain = $0.75mg(0.8)^2/(2x1.2)$ (0.2mg)	B1		
	[$\frac{1}{2} mv^2 = mg - 0.2mg$]	M1		For an equation with terms representing PE, KE and EE in linear combination
	Maximum speed is $3.96ms^{-1}$	A1	4	
	(iii) PE loss = $mg(1.2 + x)\sin30^\circ$ or $mgd\sin30^\circ$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	EE gain = $0.75mgx^2/(2x1.2)$ or $0.75mg(d - 1.2)^2/(2x1.2)$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	[$x^2 - 1.6x - 1.92 = 0$, $d^2 - 4d + 1.44 = 0$]	M1		For using PE loss = EE gain to obtain a 3 term quadratic in x or d
	Displacement is 3.6m	A1	4	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and a = v dv/dx: In the following x, y and z represent displacement from equil. pos^n, extension, and distance OP respectively.				
	[$mv\,dv/dx = mg\sin30^\circ - 0.75mg(0.8 + x)/1.2$, $mv\,dv/dy = mg\sin30^\circ - 0.75mgy/1.2$, $mv\,dv/dz = mg\sin30^\circ - 0.75mg(z - 1.2)/1.2$]	M1		For using N2 with a = v dv/dx
	$v^2/2 = -5gx^2/16 + C$ or $v^2/2 = gy/2 - 5gy^2/16 + C$ or $v^2/2 = 5gz/4 - 5gz^2/16 + C$	A1		
	[$C = 0.6g + 5g(-0.8)^2/16$ or $C = 0.6g$ or $C = 0.6g - 5g(1.2/4) + 5g(1.2)^2/16$	M1		For using $v^2(-0.8)$ or $v^2(0)$ or $v^2(1.2) = 2(g\sin30^\circ)1.2$ as appropriate
	$v^2 = (-5x^2/8 + 1.6)g$ or $v^2 = (y - 5y^2/8 + 1.2)g$ or $v^2 = (5z/2 - 5z^2/8 - 0.9)g$	A1		
	(ii) [$v_{max}^2 = 1.6g$ or $0.8g - 0.4g + 1.2g$ or $5g - 2.5g - 0.9g$]	M1		For using $v_{max}^2 = v^2(0)$ or $v^2(0.8)$ or $v^2(2)$ as appropriate
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$5x^2 - 12.8 = 0 \rightarrow x = 1.6$, $5y^2 - 8y - 9.6 = 0 \rightarrow y = 2.4$, $5z^2 - 20z + 7.2 = 0 \rightarrow z = 3.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1	8	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and SHM analysis.				
	[$m\ddot{x} = mg\sin30^\circ - 0.75mg(0.8 + x)/1.2 \rightarrow \ddot{x} = -\omega^2 x$; $v^2 = \omega^2(a^2 - x^2)$]	M1		For using N2 with $v^2 = \omega^2(a^2 - x^2)$
	$v^2 = 5g(a^2 - x^2)/8$	A1		
		M1		For using $v^2(-0.8) = 2(g\sin30^\circ)1.2$
	$v^2 = 5g(2.56 - x^2)/8$	A1		
	(ii) [$v_{max}^2 = 5g \times 2.56 \div 8$]	M1		For using $v_{max}^2 = v^2(0)$
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$2.56 - x^2 = 0 \rightarrow x = 1.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1		

4.

5 i	$[\sin\gamma = 0.96 \div 1.2]$ $\sin\gamma = 0.8$	M1 A1 [2]	For using $v_B\sin\gamma = u_B\sin\beta$
ii	$(m)2 - (m)u_B\cos\beta = (m)v_B\cos\gamma$ $2 = v_B(0.6 + 0.28 \div 1.2)$ $v_B = 2.4,\ u_B = 2$	M1 A1 M1 A1 A1 [5]	For using the principle of conservation of momentum. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. allow $u_A\cos\alpha$ (instead of 2) for A1 For eliminating u_B or v_B. Allow with cos Or $2 = 0.28u_B + 0.72u_B$
iii	$[(2 + u_B\cos\beta)e = v_B\cos\gamma]$ $(2 + 2 \text{ x } 0.28)e = 2.4 \text{ x } 0.6$ $e = \frac{9}{16}$ or 0.5625	M1 A1ft A1 [3]	For applying Newton's exp'tal law. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. ft u_B and v_B only
iv	$[(y\text{-component})^2 = 13 - 4]$ $v_A = (y\text{-component})_{\text{before}} = 3$	M1 A1 [2]	For using ½ $(m)v^2 = 6.5(m)$ **and** $(y\text{-component})^2 = v^2 - 2^2$. Allow 1 slip.

5.

For included angle marked α or for $0.8(10.5 - 8.5\cos\alpha) = 4\cos\beta$ For opposite side marked 4/0.8 (or 4) or for -- $0.8\text{x}8.5\sin\alpha = 4\sin\beta$ $8.4^2 + 6.8^2 - 2\text{x}8.4\text{x}6.8\cos\alpha = 4^2$ $\alpha = 28.1°$	M1 A1 A1 M1 A1ft A1 [6]	For triangle with two of its sides marked 0.8 x 10.5 and 0.8 x 8.5 (or 10.5 and 8.5) or for using I = Δmv in one direction. Allow B1 for omission of 0.8 Allow B1 for omission of 0.8 For using the cosine rule or for eliminating β ft 0.8 mis-used or not used