Topic X5 Variable forces and oblique collisions (Pre-TT A) [60]

1.

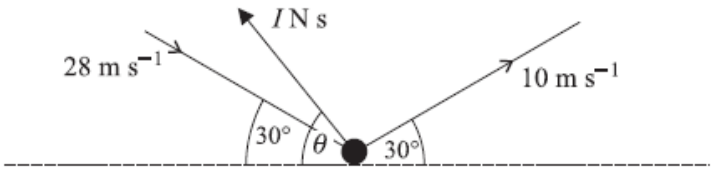
A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point A with speed $40\,\text{m s}^{-1}$ and when it has travelled a distance of x m beyond A its speed is $v\,\text{m s}^{-1}$. The engine develops a constant power of 8 kW and resistances are modelled by a force of $0.25v^2$ N opposing the motion.

(i) Show that $\dfrac{480v^2}{v^3 - 32\,000}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[5]**

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond A. **[6]**

(Total 11 marks)

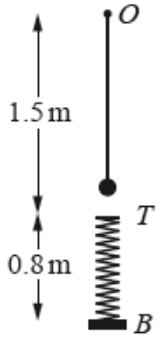
2.



When a tennis ball of mass 0.057 kg bounces it receives an impulse of magnitude I N s at an angle of θ to the horizontal. Immediately before the ball bounces it has speed $28\,\text{m s}^{-1}$ in a direction of 30° to the horizontal. Immediately after the ball bounces it has speed $10\,\text{m s}^{-1}$ in a direction of 30° to the horizontal (see diagram). Find I and θ. [7]

(Total 7 marks)

3.



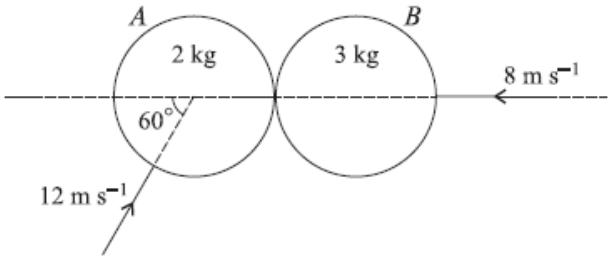
A particle of mass m kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $24mg$ N. The other end of the string is attached to a fixed point O. Directly beneath O there is a light elastic spring, of natural length 0.8 m and modulus of elasticity $32mg$ N. The bottom of the spring is attached to a fixed point B and the top of the spring is at a point T, 1.5 m vertically below O; the spring is constrained to remain vertical. The particle is projected from O with speed $0.7\,\text{m s}^{-1}$ vertically downwards. When the particle reaches T it becomes attached to the spring and it remains attached to the spring throughout the subsequent motion. The diagram shows the position as the particle first approaches T.

(i) Show that the speed of the particle at the instant when it becomes attached to the spring is $3.5\,\text{m s}^{-1}$. **[3]**

(ii) Find the distances below O at which the particle is instantaneously at rest in the subsequent motion. **[5]**

(Total 8 marks)

4.



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision A is moving with speed $12\,\text{m s}^{-1}$ at $60°$ to the line of centres, and B is moving with speed $8\,\text{m s}^{-1}$ along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [12]

(Total 12 marks)

5.

A particle, P, of mass 3 kg is moving with velocity $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude $\sqrt{130}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + \lambda\mathbf{j})\,\text{m s}^{-1}$, where λ is a positive constant.

(a) Find $\mathbf{I}$, giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$.

(6)

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

(b) Find the value of θ

(3)

(Total 9 marks)

6.

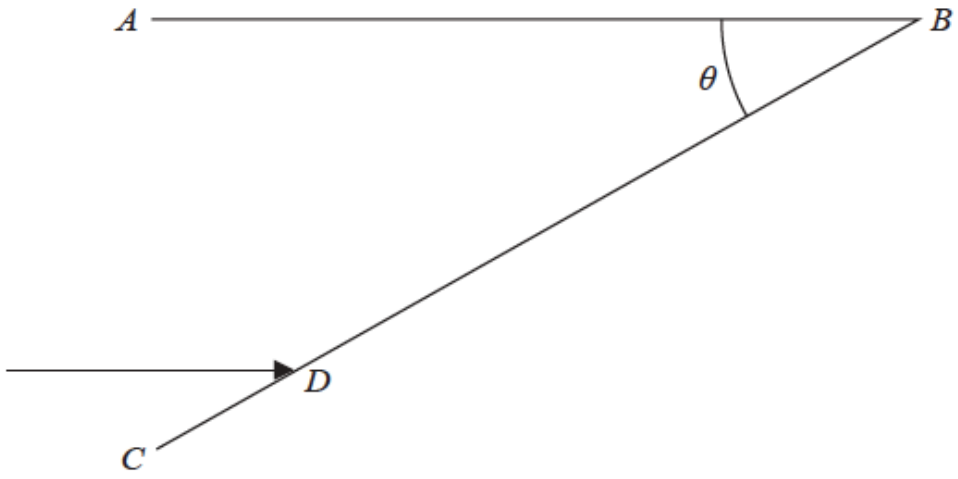


Figure 1

Figure 1 shows a plan view of two smooth fixed vertical walls, AB and BC, which are fixed to smooth horizontal ground. The angle between the walls is θ, where θ is an acute angle. A particle P is moving in a straight line on the ground and hits the wall BC at the point D.

Immediately before P hits the wall BC, P is moving in a direction that is parallel to the wall AB, as shown in Figure 1. Particle P bounces off BC and, after one impact with AB, P then hits BC for a second time. The coefficient of restitution between P and each wall is e.

Given that when P hits BC for a second time, P is moving in a direction that is perpendicular to BC,

(a) show that

$$e(e + 2)\tan^2\theta = 1$$

(10)

(b) Deduce that, for the motion to be possible, the angle between the walls must be at least 30°.

(3)

(Total 13 marks)