# Topic X5 Variable forces and oblique collisions (Pre-TT A) [60] MARKSCHEME

1.

| | | | |
|---|---|---|---|
| **4 i** | $F - 0.25v^2 = 120v(dv/dx)$<br>$F = 8000/v$<br><br>$[32000 - v^3 = 480v^2(dv/dx)]$<br>$\frac{480v^2}{v^3 - 32000}\frac{dv}{dx} = -1$ | M1<br>A1<br>B1<br><br>M1<br><br>A1<br>[5] | For using Newton's second law with $a = v(dv/dx)$<br><br><br>For substituting for $F$ and multiplying throughout by $4v$ (or equivalent)<br><br>AG |
| **ii** | $\int \frac{480v^2}{v^3 - 32000} dv = -\int dx$<br>$160 \ln(v^3 - 32000) = -x \quad (+A)$<br><br>$160 \ln(v^3 - 32000) = -x + 160 \ln 32000$<br>or<br>$160 \ln(v^3 - 32000) - 160 \ln 32000 = -500$<br><br>$(v^3 - 32000)/32000 = e^{-x/160}$<br>Speed of $m/c$ is $32.2 ms^{-1}$ | M1<br><br>A1<br><br>M1<br><br>A1ft<br><br>B1ft<br>B1<br>[6] | For separating variables and integrating<br><br><br>For using $v(0) = 40$ or $[160 \ln(v^3 - 32000)]^{v}_{40} = [-x]^{500}_{0}$<br><br>ft where factor 160 is incorrect but +ve,<br><br>Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of $A$ |

2.

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|---|---|---|---|
| $28\cos 30° - 10\cos 30°$ $[= \lvert\Delta v_H\rvert = (I/m)\cos\theta]$ | B1 | | |
| $10\sin 30° + 28\sin 30°$ $[= \lvert\Delta v_V\rvert = (I/m)\sin\theta]$ | B1 | | |
| $[X = -I\cos\theta = -0.8885, Y = I\sin\theta = 1.083]$ | M1 | | For using mv change for component or resultant |
| | M1 | | For using $I^2 = X^2 + Y^2$ |
| $I = 1.40$ | A1 | | |
| $[\tan\theta = 1.083/0.8885$ or $19/15.588..]$ | M1 | | For using $\theta = \tan^{-1}(Y/-X)$ or $\tan^{-1}(\lvert\Delta v_V\rvert / \lvert\Delta v_H\rvert)$ |
| $\theta = 50.6$ | A1 | 7 | |
| **ALTERNATIVELY** | | | |
| | M1 | | For using cosine rule in correct triangle |
| $(I/m)^2 = 28^2 + 10^2 - 2\times28\times10\cos 60° \; [=604]$ | A1 | | |
| $[I = 0.057\sqrt{604}]$ | M1 | | For using I = mv change |
| $I = 1.40$ | A1 | | |
| | M1 | | For using sine rule in correct triangle |
| $(I/m)/\sin 60° =$ $10/\sin(\theta - 30°)$ or $28/\sin(150° - \theta)$ | A1 | | |
| $\theta = 50.6$ | A1 | 7 | |

3.

| | | | | |
|---|---|---|---|---|
| (i) | $\frac{1}{2}m\times 0.7^2=\frac{1}{2}mv^2+\frac{24mg0.3^2}{2\times 1.2}-mg\times 1.5$<br><br>Speed = 3.5 ($ms^{-1}$) | M1<br><br>A1<br><br>A1 [3] | By energy; needs KE, PE and EE terms<br><br>OR $\frac{1}{2}m\times 4.9^2=\frac{1}{2}mv^2+\frac{24mg0.3^2}{2\times 1.2}-mg\times 0.3$<br>**AG** Adequate working, no errors | Allow wrong signs, missing '2' |
| (ii) | One correct EE term involving $x$ seen<br>$\frac{1}{2}m\times 0.7^2=\frac{24mg(x-1.2)^2}{2\times 1.2}+\frac{32mg(x-1.5)^2}{2\times 0.8}-mgx$<br><br>$[48x^2-136x+95=0]$<br>1.25 (m) and 1.58 (m) | B1<br>M1<br><br>A1<br><br>M1<br>A1 [5] | Where $x$ is distance below O<br>OR, where $x$ is dist from T, $\frac{1}{2}m\times 0.7^2=\frac{24mg(x+0.3)^2}{2\times 1.2}+\frac{32mgx^2}{2\times 0.8}-mg(x+1.5)$<br>Leads to $48x^2+8x-1=0$<br>Correct attempt to solve their 3 term quad.<br>$1\frac{1}{4}$ $1\frac{7}{12}$ | Energy equation with at least 1 KE, 1 PE and 1 EE term and values subst.<br>Alt left side:<br>$\frac{1}{2}m3.5^2+\frac{24mg0.3^2}{2\times 1.2}-1.5mg$<br>Dep M1 above |

4.

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|---|---|---|---|
| | M1 | | Σ mv conserved in **i** direction. |
| $2x12\cos 60^\circ - 3x8 = 2a + 3b$ | A1 | | |
| | M1 | | For using NEL |
| For LHS of equation below | A1 | | |
| $0.5(12\cos 60^\circ + 8) = b - a$ | A1 | | Complete equation with signs of a and b consistent with previous equation. |
| | M1 | | For eliminating a or b. |
| Speed of B is $0.4ms^{-1}$ in **i** direction | A1 | | |
| a = -6.6 | A1 | | |
| Component of A's velocity in **j** direction is $12\sin 60^\circ$ | B1 | | May be shown on diagram or implied in subsequent work. |
| Speed of A is $12.3ms^{-1}$ | B1ft | | |
| | M1 | | For using $\theta = \tan^{-1}(\mathbf{j}\text{comp}/\pm\mathbf{i}\text{ comp})$ |
| Direction is at 122.4° to the **i** direction | A1ft | 1<br>2 | Accept $\theta = 57.6^\circ$ with $\theta$ correctly identified. |

5.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | Impulse momentum equation | M1 | 2.1 |
| | $\mathbf{I}=3(-\mathbf{i}+\lambda\mathbf{j})-3(2\mathbf{i}+\mathbf{j})\left(=-9\mathbf{i}+(3\lambda-3)\mathbf{j}\right)$ | A1 | 1.1b |
| | Magnitude of the impulse | M1 | 1.1b |
| | $130=81+(3\lambda-3)^2$ Follow their **I** | A1ft | 1.1b |
| | $3\lambda-3=(\pm)7\ ,\quad \lambda=\frac{10}{3}$ | M1 | 2.2a |
| | $\mathbf{I}=-9\mathbf{i}+7\mathbf{j}$ (Ns) | A1 | 1.1b |
| | | **(6)** | |

| | | | |
|---|---|---|---|
| **3(b)** | Use of scalar product: $(2\mathbf{i}+\mathbf{j})\cdot\left(-\mathbf{i}+\frac{10}{3}\mathbf{j}\right)=\frac{4}{3}$ | M1 | 3.1a |
| | $\cos\theta=\frac{\frac{4}{3}}{\sqrt{\frac{109}{9}}\sqrt{5}}\left(=\frac{4}{\sqrt{545}}\right)$ follow their $\lambda$ | A1ft | 1.1b |
| | $\theta=80.1$ | A1 | 1.1b |
| | | **(3)** | |
| **3(b) alt** | Use trig to find 2 relevant angles: $\tan^{-1}\pm\frac{10}{3}$ , $\tan^{-1}\frac{1}{2}$ | (M1) | 3.1a |
| | $73.30°$ or $106.70°$, $26.57°$ | (A1) | 1.1b |
| | $\theta=80.1$ | (A1) | 1.1b |
| | | **(3)** | |
| | | | **(9 marks)** |

6.

| | | | |
|---|---|---|---|
| **7(a)** | At *D*, use CLM along wall | M1 | 3.1a |
| | $(m)u\cos\theta=(m)v\cos\alpha$ | A1 | 1.1b |
| | At *D*, use NIL along normal | M1 | 3.4 |
| | $eu\sin\theta=v\sin\alpha$ | A1 | 1.1b |
| | Overall strategy to obtain connection between angles: $e\tan\theta=\tan\alpha$ | M1 | 3.1a |
| | Use this result at *E* (second impact on *AB*) | M1 | 3.4 |
| | $e\tan(\theta+\alpha)=\tan(90^\circ-\theta)$ | A1 | 1.1b |
| | | A1 | 1.1b |
| | Expand and sub for $\tan\alpha$ | M1 | 1.1b |
| | $1=e(e+2)\tan^2\theta$ | A1* | 2.1 |
| | | **(10)** | |
| **(b)** | $0 < e\leq 1\Rightarrow\tan^2\theta\geq\frac{1}{3}$ | M1 | 2.1 |
| | $\tan\theta\geq\frac{1}{\sqrt{3}}$ | A1 | 1.1b |
| | $\theta\geq 30^o$ i.e. angle between walls must be at least 30° * | A1* | 2.2a |
| | | **(3)** | |
| | | | **(13 marks)** |