Topic X5 Variable forces and oblique collisions (Pre-TT B) [62]

1.

A light elastic string has natural length 3 m. One end is attached to a fixed point O and the other end is attached to a particle of mass 1.6 kg. The particle is released from rest in a position 5 m vertically below O. Air resistance may be neglected.

(i) Given that in the subsequent motion the particle just reaches O, show that the modulus of elasticity of the string is 117.6 N. [4]

(ii) Calculate the speed of the particle when it is 4.5 m below O. [4]

(Total 8 marks)

2.

A particle P of mass m kg is released from rest and falls vertically. When P has fallen a distance of x m it has a speed of v m s^{-1}. The only forces acting on P are its weight and air resistance of magnitude $\frac{1}{400}mv^2$ N.

(i) Find v^2 in terms of x and show that v^2 must be less than 3920. **[8]**

(ii) Find the speed of P when it has fallen 100 m. **[2]**

(Total 10 marks)

3.

A tennis ball of mass 0.057 kg has speed 10 m s^{-1}. The ball receives an impulse of magnitude 0.6 N s which reduces the speed of the ball to 7 m s^{-1}. Using an impulse-momentum triangle, or otherwise, find the angle the impulse makes with the original direction of motion of the ball. [7]

(Total 7 marks)

4.

A small ball of mass 0.5 kg is moving on a smooth horizontal plane with velocity $(4\mathbf{i} - \mathbf{j})$ m s^{-1} when it strikes a fixed vertical wall. Immediately after the impact the velocity of the ball is $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. The ball is modelled as a particle and the wall is modelled as a smooth plane surface.

(a) Find the magnitude of the impulse of the wall on the ball in the impact.

(4)

(b) Find the loss in kinetic energy of the ball due to its impact with the wall.

(3)

(c) Find the coefficient of restitution between the ball and the wall.

(5)

(d) Verify that the component of the momentum of the ball, parallel to the line of intersection of the wall and the horizontal plane, is unchanged by the impact.

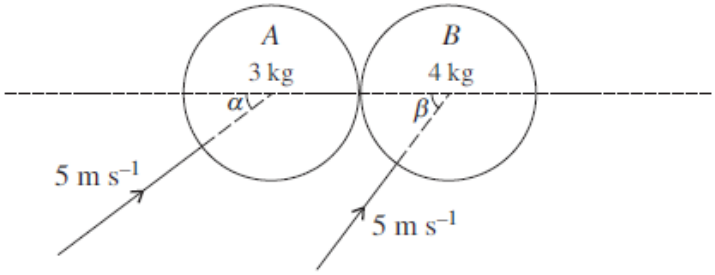
(2)

(e) State which modelling assumption ensures that the component of the momentum of the ball, parallel to the line of intersection of the wall and the horizontal plane, is unchanged by the impact.

(1)

(Total 15 marks)

5.



Two smooth uniform spheres A and B, of equal radius, have masses 3 kg and 4 kg respectively. They are moving on a horizontal surface, each with speed $5\,\text{m s}^{-1}$, when they collide. The directions of motion of A and B make angles α and β respectively with the line of centres of the spheres, where $\sin\alpha = \cos\beta = 0.6$ (see diagram). The coefficient of restitution between the spheres is 0.75. Find the angle that the velocity of A makes, immediately after impact, with the line of centres of the spheres.

[10]

6.

[*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

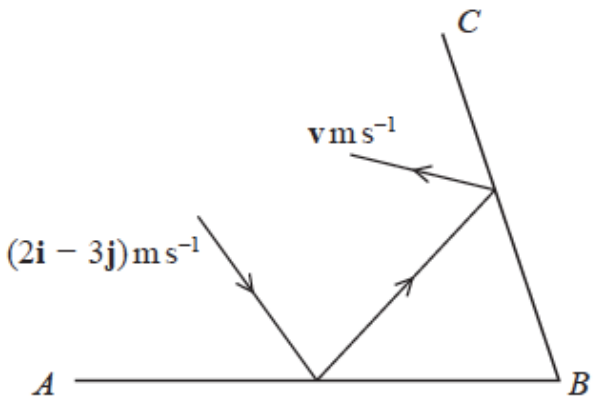


Figure 3

Figure 3 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls. The direction of $\overrightarrow{AB}$ is in the direction of the vector **i** and the direction of $\overrightarrow{BC}$ is in the direction of the vector $(-\mathbf{i} + 3\mathbf{j})$.

A small ball is projected along the floor towards wall AB so that, immediately before hitting wall AB, the velocity of the ball is $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$.

The ball hits wall AB and then hits wall BC.

The coefficient of restitution between the ball and wall AB is $\frac{1}{2}$

The coefficient of restitution between the ball and wall BC is $\frac{1}{3}$

The velocity of the ball immediately after hitting wall BC is $\mathbf{v}\,\text{m s}^{-1}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

Show that $\mathbf{v} = \left(-\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$.

(12)