Topic X5 Variable forces and oblique collisions (Pre-TT B) [62] MARKSCHEME

1.

(i)	M1		For use of EE formula
EE = λ x $(5\text{-}3)^2/(2 \times 3)$	A1		
$2\lambda/3 = 1.6 \times 9.8 \times 5$	M1		For equating EE and PE
$\lambda = 117.6$ N	A1	[4]	AG
(ii)	M1		For use of conservation of energy
$0.5\text{x}1.6\text{v}^2 = 1.6\text{x}9.8\text{x}4.5$ - $117.6\text{x}1.5^2/(2\text{x}3)$	A2,1,0		-1 each error
v = 5.75 ms^{-1}	A1	[4]	

2.

(i)	$v\,dv/dx = g - 0.0025v^2$	M1 A1	For using N's 2nd law with $a = v\,dv/dx$; 3 terms
	$\int \frac{v\,dv}{g - 0.0025v^2} = \int dx$	M1	For correctly separating variable and attempting to integrate
	$-200\ln(g - 0.0025v^2) = x\ (+A)$	A1	
	$A = -200\ln g$	M1*	Attempt to find A from $B\ln(C - Dv^2)$
	$[g - 0.0025v^2 = ge^{-0.005x}]$	*M1	For transposing equation to remove ln
	$v^2 = 400g(1 - e^{-0.005x})$	A1	
	$0 < e^{-0.005x} \le 1 \rightarrow v^2$ cannot reach $400g$ ie cannot reach 3920	B1	dependent on getting other 7 marks. Need '0 <' oe
		[8]	
(ii)	$v^2 = 400g(1 - e^{-0.5})$	M1	For substituting for x and evaluating v must have $v^2 = A + Be^{Cx}$ for (i), but not neces in this form
	Speed of P is 39.3 ms^{-1}	A1 [2]	

3.

[Magnitudes 0.6, 0.057 x 7, 0.057 x 10]	M1		For triangle with magnitudes shown
For magnitudes of 2 sides correctly marked	A1		
For magnitudes of all 3 sides correctly marked	A1		
	M1		For attempting to find angle (α) opposite to the side of magnitude 0.057 x 7
	M1		For correct use of the cosine rule or equivalent
$0.399^2 = 0.57^2 + 0.6^2 - 2 \times 0.57 \times 0.6\cos\alpha$	A1ft		
Angle is 140°	A1	7	$(180 - 39.8)°$
ALTERNATIVE METHOD			
	M1		For using I= Δmv parallel to the initial direction of motion or parallel to the impulse
$-0.6\cos\alpha = 0.057 \times 7\cos\beta - 0.057 \times 10$ or $0.6 = 0.057\text{x}10\cos\alpha + 0.057\text{x}7\cos\gamma$	A1		
	M1		For using I= Δmv perpendicular to the initial direction of motion or perpendicular to the impulse
$0.6\sin\alpha = 0.057 \times 7\sin\beta$ or $0.057\text{x}10\sin\alpha = 0.057\text{x}\,7\sin\gamma$	A1		
	M1		For eliminating β *or γ
$0.399^2 = (0.57 - 0.6\cos\alpha)^2 + (0.6\sin\alpha)^2$ or $0.399^2 = (0.6 - 0.57\cos\alpha)^2 + (0.057\sin\alpha)^2$	A1ft		
Angle is 140°	A1	7	$(180 - 39.8)°$

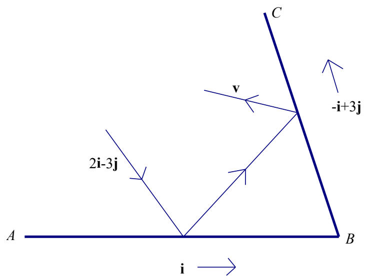
4.

Question	Scheme	Marks	AOs
5(a)	Use of Impulse-momentum principle	M1	3.1b
	$\mathbf{I} = 0.5\{(2\mathbf{i}+3\mathbf{j})-(4\mathbf{i}-\mathbf{j})\} = (-\mathbf{i}+2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{I}\rvert = \sqrt{(-1)^2+2^2}$	M1	1.1b
	$\sqrt{5}$ (N s)	A1	1.1b
		(4)	
(b)	KE Loss = Initial KE $-$Final KE	M1	3.4
	$= \frac{1}{2}\times 0.5\{(4^2+(-1)^2)-(2^2+3^2)\}$	A1	1.1b
	= 1 (J)	A1	1.1b
		(3)	
(c)	Resolve velocities along the normal (impulse)	M1	3.1b
	Separation speed = $(2\mathbf{i}+3\mathbf{j}).\frac{1}{\sqrt{5}}(-\mathbf{i}+2\mathbf{j}) = \frac{4}{\sqrt{5}}$	A1	1.1b
	Approach speed = $(4\mathbf{i}-\mathbf{j}).\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j}) = \frac{6}{\sqrt{5}}$	A1	1.1b
	Use of Newton's Impact Law along normal: $e = \frac{\frac{4}{\sqrt{5}}}{\frac{6}{\sqrt{5}}}$	M1	3.4
	$e = \frac{2}{3}$	A1	1.1b
		(5)	
(d)	Find vector along the wall $\pm(2\mathbf{i}+\mathbf{j})$ and resolve	M1	3.1a
	$0.5\times(2\mathbf{i}+3\mathbf{j}).\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}) = \frac{7}{2\sqrt{5}};\ \ 0.5\times(4\mathbf{i}-\mathbf{j}).\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}) = \frac{7}{2\sqrt{5}}$ Hence momentum conserved 'along the wall' *	A1*	2.4
		(2)	
(e)	Wall is modelled as being smooth	B1	3.5b
		(1)	
		(15 marks)	

5.

Initial **i** components of velocity for A and B are $4ms^{-1}$ and $3ms^{-1}$ respectively.	B1	May be implied.
$3x4 + 4x3 = 3a + 4b$	M1 A1	For using p.c.mmtm. parallel to l.o.c.
$0.75(4 - 3) = b - a$	M1 A1	For using NEL
$a = 3$	M1 A1	For attempting to find a Depends on all three M marks
Final **j** component of velocity for A is $3ms^{-1}$	B1	May be implied
Angle with l.o.c. is 45° or 135°	M1 A1ft [10]	For using $\tan^{-1}(v_j/v_i)$ for A ft incorrect value of a ($\neq 0$) only
		SR for consistent sin/cos mix (max 8/10) $3x3 + 4x4 = 3a + 4b$ and $b - a = 0.75(3 - 4)$ M1 M1 as scheme and A1 for *both* equ's $a = 4$ M1 as scheme A1 j component for A is $4ms^{-1}$ B1 Angle $\tan^{-1}(4/4) = 45°$ M1 as scheme A1

6.

Question	Scheme	Marks	AOs
5			
	After first impact: parallel to *AB* 2**i**	B1	2.1
	Use of impact law perpendicular to *AB*	M1	3.4
	$-\frac{1}{2}(-3\mathbf{j}) = \frac{3}{2}\mathbf{j}$	A1	1.1b
	Strategy to find final velocity	M1	3.1b
	Second impact: parallel to *BC* $\mathbf{v}\cdot\left(\frac{-\mathbf{i}+3\mathbf{j}}{(\sqrt{10})}\right)$	M1	3.1b
	$\left(\left(2\mathbf{i}+\frac{3}{2}\mathbf{j}\right)\cdot\left(\frac{-\mathbf{i}+3\mathbf{j}}{(\sqrt{10})}\right)=\frac{5}{2\sqrt{10}}\right)$ follow their **v**	A1ft	1.1b
	Component of velocity $=\frac{5}{2\sqrt{10}}\times\left(\frac{-\mathbf{i}+3\mathbf{j}}{\sqrt{10}}\right)=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})$	A1	1.1b
	Vector perpendicular to the wall $(3\mathbf{i}+\mathbf{j})$	B1	3.1b
	Use of impact law:	M1	3.4

	$-\frac{1}{3}\left(2\mathbf{i}+\frac{3}{2}\mathbf{j}\right)\cdot\frac{(3\mathbf{i}+\mathbf{j})}{\left(\sqrt{10}\right)}$ Follow their velocity and their perpendicular vector	A1ft	1.1b
	Component of velocity $=-\frac{5}{2\sqrt{10}}\times\left(\frac{3\mathbf{i}+\mathbf{j}}{\sqrt{10}}\right)=-\frac{1}{4}(3\mathbf{i}+\mathbf{j})$	A1	1.1b
	$\Rightarrow \mathbf{v}=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})-\frac{1}{4}(3\mathbf{i}+\mathbf{j})$ (sum of their components)		
	$=\left(-\mathbf{i}+\frac{1}{2}\mathbf{j}\right)$ (m s^{-1}) *	A1*	2.2a
		(12)	
5 alt	***For the last 9 marks***		
	Strategy to find final velocity	M1	
	Perpendicular to $-\mathbf{i}+3\mathbf{j}$ is $-3\mathbf{i}-\mathbf{j}$	B1	
	Find components of the initial velocity parallel and perpendicular to $-\mathbf{i}+3\mathbf{j}$: $\mathbf{v}=p(-\mathbf{i}+3\mathbf{j})+q(-3\mathbf{i}-\mathbf{j})$	M1	
	$\begin{cases}2=-p-3q\\ \frac{3}{2}=3p-q\end{cases}\Rightarrow p=\frac{1}{4}$	A1	
	$q=-\frac{3}{4},\ \left(\mathbf{v}=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})-\frac{3}{4}(-3\mathbf{i}-\mathbf{j})\right)$	A1	
	Impact law perpendicular to plane: $\pm\frac{1}{3}\times-\frac{3}{4}(-3\mathbf{i}-\mathbf{j})$	M1	
	Follow their perpendicular component	A1ft	
	Parallel component: $\frac{1}{4}(-\mathbf{i}+3\mathbf{j})$ Follow their parallel component	A1ft	
	Final velocity $=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})+\frac{1}{4}(-3\mathbf{i}-\mathbf{j})=-\mathbf{i}+\frac{1}{2}\mathbf{j}$ *	A1*	
			(12 marks)