

Topic Z1 Vectors (Post-TT A) [43]

1.

The line l_1 passes through the points $(0, 0, 10)$ and $(7, 0, 0)$ and the line l_2 passes through the points $(4, 6, 0)$ and $(3, 3, 1)$. Find the shortest distance between l_1 and l_2 . [7]

(Total 7 marks)

2.

A line l has equation $\frac{x-6}{-4} = \frac{y+7}{8} = \frac{z+10}{7}$ and a plane p has equation $3x - 4y - 2z = 8$.

(i) Find the point of intersection of l and p . [3]

(ii) Find the equation of the plane which contains l and is perpendicular to p , giving your answer in the form $ax + by + cz = d$. [5]

(Total 8 marks)

3.

The plane Π passes through the points $(1, 2, 1)$, $(2, 3, 6)$ and $(4, -1, 2)$.

(i) Find a cartesian equation of the plane Π . [5]

The line l has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$.

(ii) Find the coordinates of the point of intersection of Π and l . [3]

(iii) Find the acute angle between Π and l . [3]

(Total 11 marks)

4.

Three planes Π_1 , Π_2 and Π_3 have equations

$$\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 5, \quad \mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 6, \quad \mathbf{r} \cdot (\mathbf{i} + 5\mathbf{j} - 12\mathbf{k}) = 12,$$

respectively. Planes Π_1 and Π_2 intersect in a line l ; planes Π_2 and Π_3 intersect in a line m .

(i) Show that l and m are in the same direction. [5]

(ii) Write down what you can deduce about the line of intersection of planes Π_1 and Π_3 . [1]

(iii) By considering the cartesian equations of Π_1 , Π_2 and Π_3 , or otherwise, determine whether or not the three planes have a common line of intersection. [4]

(Total 10 marks)

5.

The line l_1 has equation $\frac{x-2}{4} = \frac{y-4}{-2} = \frac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π .

Find a vector equation of the line l_2

(7)

(Total 7 marks)