Topic Z1 Vectors (Post-TT A) [43] MARKSCHEME

1.

Direction of $l_1 = k[7, 0, -10]$ Direction of $l_2 = k[1, 3, -1]$	B1	For both directions
EITHER $\mathbf{n} = [7, 0, -10] \times [1, 3, -1]$ *OR* $[x, y, z].[7, 0, -10] = 0 \Rightarrow 7x - 10z = 0$; $[x, y, z].[1, 3, -1] = 0 \Rightarrow x + 3y - z = 0$	M1	For finding vector product of directions of l_1 and l_2 *OR* for using 2 scalar products and obtaining equations
$\Rightarrow \mathbf{n} = k[10, -1, 7]$	A1	For correct $\mathbf{n}$
METHOD 1 Vector $(\mathbf{a} - \mathbf{b})$ from l_1 to $l_2 = \pm[4, 6, -10]$ *OR* $\pm[-4, 3, 1]$ *OR* $\pm[3, 3, -9]$ *OR* $\pm[-3, 6, 0]$	B1	For a correct vector
$d = \dfrac{\lvert(\mathbf{a} - \mathbf{b}).\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert} = \dfrac{36}{\sqrt{150}}$	M1*	For finding $(\mathbf{a} - \mathbf{b}).\mathbf{n}$
	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
$d = \frac{6}{5}\sqrt{6} \approx 2.94$	A1 7	For correct distance **AEF**
METHOD 2 Planes containing l_1 and l_2 perp. to $\mathbf{n}$	M1*	For finding planes and $p_1 - p_2$ seen
are $\mathbf{r}.[10, -1, 7] = p_1 = 70$, $\mathbf{r}.[10, -1, 7] = p_2 = 34$	B1	For $p_1 = 70k$ and $p_2 = 34k$
$\Rightarrow d = \dfrac{\lvert 70 - 34 \rvert}{\sqrt{150}} = \dfrac{36}{\sqrt{150}} = \frac{6}{5}\sqrt{6} \approx 2.94$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
	A1	For correct distance **AEF**
METHOD 3 $\mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda]$ *OR* $[7 + 7\lambda, 0, -10\lambda]$ $\mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu]$ *OR* $[3 + \mu, 3 + 3\mu, 1 - \mu]$	B1	For correct points on l_1 and l_2 using different parameters
$7\lambda + 10\alpha - \mu = 4 \mid -3 \mid 3 \mid -4$ $-\alpha - 3\mu = 6 \mid 6 \mid 3 \mid 3$ $-10\lambda + 7\alpha + \mu = -10 \mid 0 \mid -9 \mid 1$ $\Rightarrow \alpha = -\frac{6}{25}$	M1*	For setting up 3 linear equations from $\mathbf{r}_1 + \alpha\mathbf{n} = \mathbf{r}_2$ and solving for α
$\lvert\mathbf{n}\rvert = \sqrt{150}$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ seen multiplying α
$\Rightarrow d = \frac{6}{25}\sqrt{150} = \frac{6}{5}\sqrt{6} \approx 2.94$	A1	For correct distance **AEF**
	[7]	

2.

(i)	$[6-4\lambda, -7+8\lambda, -10+7\lambda]$ on p	B1	For point on l seen or implied
	$\Rightarrow 3(6-4\lambda)-4(-7+8\lambda)-2(-10+7\lambda)=8$	M1	For substituting into equation of p
	$\Rightarrow \lambda = 1 \Rightarrow (2, 1, -3)$	A1 **3**	For correct point. Allow position vector
(ii)	METHOD 1		
	$\mathbf{n}=[-4, 8, 7]\times[3, -4, -2]$	M1*	For direction of l and normal of p seen
		M1 (*dep)	For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$
	$\mathbf{n}=k[12, 13, -8]$	A1	For correct vector
	$(2, 1, -3)$ OR $(6, -7, -10)$	M1	For finding scalar product of their point on l with their attempt at $\mathbf{n}$, or equivalent
	$\Rightarrow 12x+13y-8z=61$	A1 **5**	For correct equation, aef cartesian
	METHOD 2		
	$\mathbf{r}=[2, 1, -3]$ *OR* $[6, -7, -10]$ $+\lambda[-4, 8, 7]+\mu[3, -4, -2]$	M1 A1√	For stating eqtn of plane in parametric form (may be implied by next stage), using $[2, 1, -3]$ (ft from (i)) Or $[6, -7, -10]$, $\mathbf{n}_1$ and $\mathbf{n}_2$ (as above)
	$x = 2-4\lambda+3\mu$	M1	For writing as 3 linear equations
	$y = 1+8\lambda-4\mu$	M1	For attempting to eliminate λ and μ
	$z=-3+7\lambda-2\mu$		
	$\Rightarrow 12x+13y-8z=61$	A1	For correct equation aef cartesian
	METHOD 3		
	$3(6+3\mu)-4(-7-4\mu)-2(-10-2\mu)=8$	M1	For finding foot of perpendicular from point on l to p
	$\Rightarrow \mu=-2 \Rightarrow (0, 1, -6)$	A1	For correct point or position vector
	From 3 points $(2, 1, -3)$, $(6, -7, -10)$, $(0, 1, -6)$,		
	$\mathbf{n}$ = vector product of 2 of		
	$[2, 0, 3]$, $[6, -8, -4]$, $[-4, 8, 7]$	M1	Use vector product of 2 vectors in plane
	$\Rightarrow \mathbf{n}=k[12, 13, -8]$		
	$(2, 1, -3)$ OR $(6, -7, -10)$	M1	For finding scalar product of their point on l with their attempt at $\mathbf{n}$, or equivalent
	$\Rightarrow 12x+13y-8z=61$	A1	For correct equation aef cartesian
		8	

3.

3	**(i)**		vectors in plane $\begin{pmatrix}2\\3\\6\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}1\\1\\5\end{pmatrix}$ and $\begin{pmatrix}4\\-1\\2\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}3\\-3\\1\end{pmatrix}$	M1*	or multiple(s)	or multiple of $\begin{pmatrix}-2\\4\\4\end{pmatrix}$
			$\begin{pmatrix}1\\1\\5\end{pmatrix}\times\begin{pmatrix}3\\-3\\1\end{pmatrix}=\begin{pmatrix}16\\14\\-6\end{pmatrix}=2\begin{pmatrix}8\\7\\-3\end{pmatrix}$	M1dep* A1	for **M1**, method shown or 2 correct elements	
			$\mathbf{r}\cdot\begin{pmatrix}8\\7\\-3\end{pmatrix}=\begin{pmatrix}1\\2\\1\end{pmatrix}\cdot\begin{pmatrix}8\\7\\-3\end{pmatrix}$	M1		
			$8x+7y-3z=19$	A1 [5]	AEF (Cartesian)	
3	**(ii)**		$x=-1+4\lambda, y=-2+3\lambda, z=6-2\lambda$			
			$8(-1+4\lambda)+7(-2+3\lambda)-3(6-2\lambda)=19$ $\Rightarrow \lambda=1$	M1 M1	solves and attempts substitution	
			intersect at (3, 1, 4)	A1 [3]	Accept vector form	
3	**(iii)**		$\cos(\alpha)=\dfrac{\left\lvert\begin{pmatrix}8\\7\\-3\end{pmatrix}\cdot\begin{pmatrix}4\\3\\-2\end{pmatrix}\right\rvert}{\sqrt{8^2+7^2+3^2}\sqrt{4^2+3^2+2^2}}=\dfrac{59}{\sqrt{122}\sqrt{29}}$	M1*		can be implied by 7.3° or 0.13 or $\cos\alpha=0.9919$ seen
			$\theta=\frac{1}{2}\pi-\alpha$	M1dep*	can use $\sin\theta$	consistent use of degrees or radians
			$\theta\approx1.44$ or $\theta\approx82.7°$	A1 [3]		

4.

(i)	$[1, 1, -2] \times [1, -1, 3] = (\pm)[1, -5, -2]$	M1 A1		For using × of direction vectors For correct direction
	$[1, -1, 3] \times [1, 5, -12] = (\pm)[-3, 15, 6]$	M1 A1		For using × of direction vectors For correct direction
	$[-3, 15, 6] = k[1, -5, -2] \Rightarrow$ parallel	A1	5	For argument completed **AG** ($k = -3$ not essential)
(ii)	Line of intersection is parallel to l and m	B1	1	For correct statement
(iii)	METHOD 1			
	$\left.\begin{matrix} x+y-2z=5 \\ x-y+3z=6 \end{matrix}\right\}$ e.g. $z=0 \Rightarrow \left(\frac{11}{2}, -\frac{1}{2}, 0\right)$ on l	M1 A1		For attempt to find points on 2 lines For a correct point on one line
	$\left.\begin{matrix} x-y+3z=6 \\ x+5y-12z=12 \end{matrix}\right\}$ e.g. $z=0 \Rightarrow (7, 1, 0)$ on m	A1		For a correct point on another line
	$\left.\begin{matrix} x+y-2z=5 \\ x+5y-12z=12 \end{matrix}\right\}$ e.g. $z=0 \Rightarrow \left(\frac{13}{4}, \frac{7}{4}, 0\right)$ on l_3			
	Different points $\Rightarrow$ no common line of intersection	A1	4	For correct answer
	METHOD 2			
	$\left.\begin{matrix} x+y-2z=5 \\ x-y+3z=6 \end{matrix}\right\}$ e.g. $\Rightarrow z = 11-2x,\ y = 27-5x$	M1 A1		For finding (e.g.) y and z in terms of x *OR* eliminating one variable For correct expressions *OR* equations
	LHS of eqn 3 = $x + (135-25x) - (132-24x) = 3 \neq 12$	A1		For obtaining a contradiction from 3rd equation
	$\Rightarrow$ no common line of intersection	A1		For correct answer
	METHOD 3			
	LHS $\Pi_3 = 3\Pi_1 - 2\Pi_2$	M2		For attempt to link 3 equations
	RHS $3\times5 - 2\times6 = 3 \neq 12$	A1		For obtaining a contradiction
	$\Rightarrow$ no common line of intersection	A1		For correct answer
	SR Variations on all methods may gain full credit			SR f.t. may be allowed from relevant working
		10		

5.

$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=\ldots$	M1	1.1b
$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10, 0, -4)$	A1	1.1b
$2+t-2(4-2t)-6+t=6 \Rightarrow t=\ldots$	M1	3.1a
$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1), 4+6(-2), -6+6(1))$	M1	3.1a
$(8, -8, 0)$	A1	1.1b
$\begin{pmatrix}10\\0\\-4\end{pmatrix} - \begin{pmatrix}8\\-8\\0\end{pmatrix} = \begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
$\mathbf{r} = \begin{pmatrix}10\\0\\-4\end{pmatrix} + k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r} - \begin{pmatrix}10\\0\\-4\end{pmatrix}\right) \times \begin{pmatrix}1\\4\\-2\end{pmatrix} = 0$	A1	2.5
	(7)	

(7 marks)