Topic Z1 Vectors (Post-TT B) [49]

1.

Two intersecting lines, lying in a plane p, have equations

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1} = \frac{y-3}{2} = \frac{z-4}{4}.$$

(i) Obtain the equation of p in the form $2x - y + z = 3$. **[3]**

(ii) Plane q has equation $2x - y + z = 21$. Find the distance between p and q. **[3]**

(Total 6 marks)

2.

A tetrahedron $ABCD$ is such that AB is perpendicular to the base BCD. The coordinates of the points A, C and D are $(-1, -7, 2)$, $(5, 0, 3)$ and $(-1, 3, 3)$ respectively, and the equation of the plane BCD is $x + 2y - 2z = -1$.

(i) Find, in either order, the coordinates of B and the length of AB. [5]

(ii) Find the acute angle between the planes ACD and BCD. [6]

(Total 11 marks)

3.

Lines l_1 and l_2 have equations

$$\frac{x-3}{2} = \frac{y-4}{-1} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-5}{4} = \frac{y-1}{3} = \frac{z-1}{2}$$

respectively.

(i) Find the equation of the plane Π_1 which contains l_1 and is parallel to l_2, giving your answer in the form $\mathbf{r.n} = p$. [5]

(ii) Find the equation of the plane Π_2 which contains l_2 and is parallel to l_1, giving your answer in the form $\mathbf{r.n} = p$. [2]

(iii) Find the distance between the planes Π_1 and Π_2. [2]

(iv) State the relationship between the answer to part **(iii)** and the lines l_1 and l_2. [1]

(Total 10 marks)

4.

The planes Π_1 and Π_2 have equations

$$\mathbf{r}.\begin{pmatrix}1\\2\\1\end{pmatrix}=3 \quad \text{and} \quad \mathbf{r}.\begin{pmatrix}2\\1\\4\end{pmatrix}=5$$

respectively. They intersect in the line l.

(i) Find cartesian equations of l. **[4]**

The plane Π_3 has equation $\mathbf{r}.\begin{pmatrix}1\\5\\-1\end{pmatrix}=1$.

(ii) Show that Π_3 is parallel to l but does not contain it. **[3]**

(iii) Verify that $(2,0,1)$ lies on planes Π_1 and Π_3. Hence write down a vector equation of the line of intersection of these planes. **[3]**

(Total 10 marks)

5.

With respect to the origin O, the position vectors of the points U, V and W are $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively. The mid-points of the sides VW, WU and UV of the triangle UVW are M, N and P respectively.

(i) Show that $\overrightarrow{UM}=\frac{1}{2}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$. **[2]**

(ii) Verify that the point G with position vector $\frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$ lies on UM, and deduce that the lines UM, VN and WP intersect at G. **[5]**

(iii) Write down, in the form $\mathbf{r}=\mathbf{a}+t\mathbf{b}$, an equation of the line through G which is perpendicular to the plane UVW. (It is not necessary to simplify the expression for $\mathbf{b}$.) **[2]**

(iv) It is now given that $\mathbf{u}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{v}=\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\mathbf{w}=\begin{pmatrix}0\\0\\1\end{pmatrix}$. Find the perpendicular distance from O to the plane UVW. **[3]**

(Total 12 marks)