Topic Z1 Vectors (Post-TT B) [49] MARKSCHEME

1.

(i)	$\mathbf{n} = [2, 1, -3] \times [-1, 2, 4]$	M1		For using $\times$ of direction vectors
	$= [10, -5, 5] = k[2, -1, 1]$	A1		For correct $\mathbf{n}$
	$(1, 3, 4) \Rightarrow 2x - y + z = 3$	A1	3	For substituting $(1, 3, 4)$ and obtaining **AG** (Verification only M0)
(ii)	METHOD 1 distance $= \dfrac{21-3}{\lvert\mathbf{n}\rvert}$ *OR* $\dfrac{\lvert[1, 3, 4].[2, -1, 1] - 21\rvert}{\lvert\mathbf{n}\rvert}$	M1		For $21 - 3$ *OR* $[1, 3, 4].[2, -1, 1] - 21$ *OR* $\lvert([1, 3, 4] - [a, b, c]).[2, -1, 1]\rvert$ soi
	OR $\dfrac{\lvert([1, 3, 4] - [a, b, c]).[2, -1, 1]\rvert}{\lvert\mathbf{n}\rvert}$ where (a, b, c) is on q	B1		For $\lvert\mathbf{n}\rvert = \sqrt{6}$ soi
	$= \dfrac{18}{\sqrt{6}} = 3\sqrt{6}$	A1	3	For correct distance **AEF**
	METHOD 2 $[1+2t, 3-t, 4+t]$ on q	M1		For forming and solving an equation in t
	$\Rightarrow 2(1+2t) - (3-t) + (4+t) = 21 \Rightarrow t = 3$	B1		For $\lvert\mathbf{n}\rvert = \sqrt{6}$ soi
	$\Rightarrow$ distance $= 3\lvert\mathbf{n}\rvert = 3\sqrt{6}$	A1		For correct distance **AEF**
	METHOD 3 As Method 2 to $t = 3 \Rightarrow (7, 0, 7)$ on q	M1*		For finding point where normal meets q
	distance from $(1, 3, 4)$	M1 (*dep)		For finding distance from $(1, 3, 4)$
	$= \sqrt{(7-1)^2 + (0-3)^2 + (7-4)^2} = \sqrt{54} = 3\sqrt{6}$	A1		For correct distance **AEF**
		6		

2.

6 (i)		*Either coordinates or vectors may be used* Methods 1 and 2 may be combined, for a maximum of 5 marks
METHOD 1 State $B = (-1, -7, 2) + t(1, 2, -2)$	M1	For using vector normal to plane
On plane $\Rightarrow (-1+t) + 2(-7+2t) - 2(2-2t) = -1$	M1 M1	For substituting parametric form into plane For solving a linear equation in t
$\Rightarrow t = 2 \Rightarrow B = (1, -3, -2)$	A1	For correct coordinates
$AB = \sqrt{2^2 + 4^2 + 4^2}$ *OR* $2\sqrt{1^2 + 2^2 + 2^2} = 6$	A1 5	For correct length of AB
METHOD 2 $AB = \left\lvert\dfrac{-1-14-4+1}{\sqrt{1^2+2^2+2^2}}\right\rvert = 6$	M1 A1	For using a correct distance formula For correct length of AB
OR $AB = \mathbf{AC}.\hat{\mathbf{AB}} = \dfrac{[6, 7, 1].[1, 2, -2]}{\sqrt{1^2+2^2+2^2}} = 6$		
$B = (-1, -7, 2) \pm 6\dfrac{(1, 2, -2)}{\sqrt{1^2+2^2+2^2}}$	M1	For using $B = A$ + length of $AB \times$ unit normal
$B = (-1, -7, 2) \pm (2, 4, -4)$	B1	For checking whether + or – is needed (substitute into plane equation)
$B = (1, -3, -2)$	A1	For correct coordinates (allow even if B0)
(ii) Find vector product of any two of $\pm[6, 7, 1]$, $\pm[6, -3, 0]$, $\pm(0, 10, 1)$	M1	For finding vector product of two relevant vectors
Obtain $k[1, 2, -20]$	A1	For correct vector $\mathbf{n}$
$\theta = \cos^{-1}\dfrac{\lvert[1, 2, -2].[1, 2, -20]\rvert}{\sqrt{1^2+2^2+2^2}\sqrt{1^2+2^2+20^2}}$	M1* M1 (dep*)	For using scalar product of two normal vectors For stating both moduli in denominator
$\theta = \cos^{-1}\dfrac{45}{\sqrt{9}\sqrt{405}} = 41.8^\circ$ (41.810...°, 0.72972...)	A1√ A1 6	For correct scalar product. f.t. from $\mathbf{n}$ For correct angle
	11	

3.

6 **(i)** $\mathbf{n} = l_1 \times l_2$	B1	For stating or implying in (i) or (ii) that **n** is perpendicular to l_1 and l_2
$\mathbf{n} = [2, -1, 1] \times [4, 3, 2]$	M1*	For finding vector product of direction vectors
$\mathbf{n} = k[-1, 0, 2]$	A1	For correct vector (any k)
$[3, 4, -1] . k[-1, 0, 2] = -5k$	M1 (*dep)	For substituting a point of l_1 into $\mathbf{r}.\mathbf{n}$
$\mathbf{r}.[-1, 0, 2] = -5$	A1 5	For obtaining correct p. **AEF** in this form
(ii) $[5, 1, 1] . k[-1, 0, 2] = -3k$	M1	For using same **n** and substituting a point of l_2
$\mathbf{r}.[-1, 0, 2] = -3$	A1√ 2	For obtaining correct p. **AEF** in this form f.t. on incorrect **n**
(iii) $d = \frac{\|-5+3\|}{\sqrt{5}}$ *OR* $d = \frac{\|[2, -3, 2].[-1, 0, 2]\|}{\sqrt{5}}$	M1	For using a distance formula from their equations Allow omission of | |
OR d from $(5, 1, 1)$ to $\Pi_1 = \frac{\|5(-1)+1(0)+1(2)+5\|}{\sqrt{5}}$		
OR d from $(3, 4, -1)$ to $\Pi_2 = \frac{\|3(-1)+4(0)-1(2)+3\|}{\sqrt{5}}$		
OR $[3-t, 4, -1+2t].[-1, 0, 2] = -3 \Rightarrow t = \frac{2}{5}$ *OR* $[5-t, 1, 1+2t].[-1, 0, 2] = -5 \Rightarrow t = -\frac{2}{5}$		*OR* For finding intersection of $\mathbf{n}_1$ and Π_2 or $\mathbf{n}_2$ and Π_1
$d = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} = 0.894427...$	A1√ 2	For correct distance **AEF** f.t. on incorrect **n**
(iv) d is the shortest *OR* perpendicular distance between l_1 and l_2	B1 1 **10**	For correct statement

4.

6	**(i)**		$\begin{pmatrix}1\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}7\\-2\\-3\end{pmatrix}$	M1A1		
			finds point on both planes	B1	e.g. (0,1,1)	or $\left(\frac{7}{3}, \frac{1}{3}, 0\right)$ or $\left(\frac{7}{2}, 0, -\frac{1}{2}\right)$
			$\frac{x}{-7} = \frac{y-1}{2} = \frac{z-1}{3}$	A1 [4]	oe	
	ALT		$x + 2y + z = 3$ $2x + y + 4z = 5$			
			$3x + 7z = 7$	M1 A1	Attempts to find at least 1 equation 2 correct equations	
			$2x + 7y = 7$			or $3y - 2z = 1$
			$\frac{x}{-7} = \frac{y-1}{2} = \frac{z-1}{3}$	M1A1 [4]	oe of the form f(x) = g(y)= h(z)	
	(ii)		$\begin{pmatrix}-7\\2\\3\end{pmatrix} \cdot \begin{pmatrix}1\\5\\-1\end{pmatrix} = -7 + 10 - 3 = 0$	M1	For scalar product, either shows method or gives answer of zero	
			$\Rightarrow l \parallel \Pi_3$	A1	for A1 must have working out line for scalar product	
			$(0, 1, 1)$ is on line, but $\begin{pmatrix}0\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\5\\-1\end{pmatrix} = 4 \neq 1$			
			so not on plane	B1 [3]		
	ALT		$x + 5y - z = 1$ $7\lambda + 5(1 - 2\lambda) - (1 - 3\lambda) = 1$	M1A1		
			$\Rightarrow 4 = 1$ inconsistent, so l is parallel and not on plane	A1 [3]		
	(iii)		$2 + 2 \times 0 + 1 = 3$ (so on Π_1)			must show working for at least one plane
			$\begin{pmatrix}2\\0\\1\end{pmatrix} \cdot \begin{pmatrix}1\\5\\-1\end{pmatrix} = 1$ (so on Π_3)	B1	Verify both	
			Line has equation $\mathbf{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}-7\\2\\3\end{pmatrix}$	M1 A1 [3]	oe vector form in cartesian form M1 only	if cross product calculated incorrectly then M0A0

5.

(i)		$\mathbf{m} = \mathbf{v} + \frac{1}{2}(\mathbf{w} - \mathbf{v}) \Rightarrow$	M1	For using vector triangle, or equivalent, for M	$\overline{UM} = \overline{UV} + \overline{VM}$ $= (\mathbf{v} - \mathbf{u}) + \frac{1}{2}(\mathbf{w} - \mathbf{v})$
		$\overrightarrow{UM} = \mathbf{v} + \frac{1}{2}(\mathbf{w} - \mathbf{v}) - \mathbf{u} = \frac{1}{2}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$	A1 [2]	For correct expression **AG** **SR** Allow use of ratio theorem	Minimum $-\mathbf{u} + \frac{1}{2}(\mathbf{v} + \mathbf{w})$
(ii)		METHOD 1 (first 3 marks) $\overrightarrow{UM}$ is $\mathbf{r} = \mathbf{u} + \frac{1}{2}t(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ $t = \frac{2}{3} \Rightarrow \mathbf{u} + \frac{1}{3}(\mathbf{v} + \mathbf{w} - 2\mathbf{u}) = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$	M1* M1* A1	For equation of UM For attempt to find a suitable value of t For $t = \frac{2}{3}$ and G obtained **AG**	
		METHOD 2 (first 3 marks) $\overrightarrow{UG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) - \mathbf{u} = \frac{1}{3}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ OR $\overrightarrow{MG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) - \frac{1}{2}(\mathbf{v} + \mathbf{w}) = -\frac{1}{6}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ $\Rightarrow U, G, M$ collinear	M1* M1* A1	For finding directions of UG or MG For comparing with UM For showing G lies on UM **AG**	
		By symmetry of $\overrightarrow{OG}$ in $\mathbf{u}, \mathbf{v}, \mathbf{w}$ G also lies on VN, WP $\Rightarrow UM, VN, WP$ intersect at G	B1 B1dep * [5]	For use of symmetry, or by repeating method for UM twice more. For complete reasoning to **AG**	
(iii)		Line is $\mathbf{r} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) + t(\mathbf{u} - \mathbf{v}) \times (\mathbf{u} - \mathbf{w})$ (etc)	B1 B1 [2]	For $r = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) + t \times$"any vector" For a correct $\mathbf{n}$, using any 2 of $\pm(\mathbf{u} - \mathbf{v})$, $\pm(\mathbf{v} - \mathbf{w})$, $\pm(\mathbf{w} - \mathbf{u})$	Allow $\overline{UV} \times \overline{VW}$ or similar
(iv)		METHOD 1 $\mathbf{n} = [1, 0, -1] \times [0, 1, -1]$ (etc) $= k[1, 1, 1]$ UVW is $\mathbf{r}.\mathbf{n} = [1, 0, 0].[1, 1, 1] = 1$ $\Rightarrow d = \frac{1}{\sqrt{3}}$	M1* M1dep * A1 [3]	For attempt to find $\mathbf{n}$ For substituting a point For correct d	May see use of $\frac{\lvert p.n - d \rvert}{\lvert n \rvert}$
		METHOD 2 UVW is $x + y + z = 1$ (from given $\mathbf{u}, \mathbf{v}, \mathbf{w}$) $\Rightarrow d = \frac{1}{\sqrt{3}}$	M2 A1	For attempt to find cartesian equation For correct d	
		METHOD 3 $\overrightarrow{OG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$ $\Rightarrow OG = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}}$ $\Rightarrow d = \frac{1}{\sqrt{3}}$	M1* M1dep * A1	For stating or implying $\lvert\overrightarrow{OG}\rvert$ is d For finding magnitude For correct d	

5.