

Topic Z1 Vectors (Pre-TT A) [49]

1.

The plane Π_1 has vector equation

$$\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point $(6, 2, 12)$ to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2

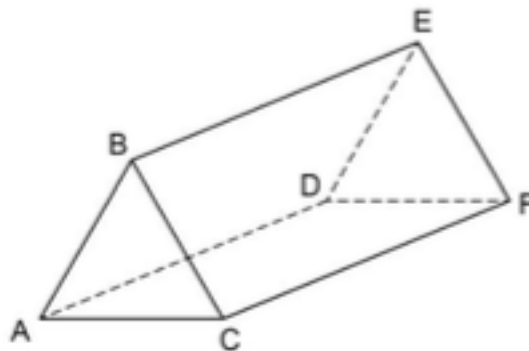
(2)

(c) Show that the acute angle between Π_1 and Π_2 is 52° to the nearest degree.

(3)

2.

A designer is using a computer aided design system to design part of a building. He models part of a roof as a triangular prism ABCDEF with parallel triangular ends ABC and DEF, and a rectangular base ACFD. He uses the metre as the unit of length.



The coordinates of B, C and D are $(3, 1, 11)$, $(9, 3, 4)$ and $(-4, 12, 4)$ respectively.

He uses the equation $x - 3y = 0$ for the plane ABC.

He uses $\left[\mathbf{r} - \begin{pmatrix} -4 \\ 12 \\ 4 \end{pmatrix} \right] \times \begin{pmatrix} 4 \\ -12 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ for the equation of the line AD.

Find the volume of the space enclosed inside this section of the roof.

[9 marks]

3.

The line l has equations $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-7}{5}$. The plane Π has equation $4x - y - z = 8$.

(i) Show that l is parallel to Π but does not lie in Π . [3]

(ii) The point $A(1, -2, 7)$ is on l . Write down a vector equation of the line through A which is perpendicular to Π . Hence find the position vector of the point on Π which is closest to A . [4]

(iii) Hence write down a vector equation of the line in Π which is parallel to l and closest to it. [1]

4.

The line L has Cartesian equations $x - p = \frac{y+2}{q} = 3 - z$ and the plane π has

equation $\mathbf{r} \cdot \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + 3 = 0$

(a) In the case where the plane fully contains the line, find the values of p and q . [3 marks]

(b) In the case where the line intersects the plane at a single point, find the range of values of p and q . [3 marks]

(c) In the case where the angle θ between the line and the plane satisfies $\sin \theta = \frac{1}{\sqrt{6}}$ and the line intersects the plane at $z = 0$

(c) (i) Find the value of q . [4 marks]

(c) (ii) Find the value of p . [3 marks]

5.

Three planes have equations,

$$\begin{aligned}x - y + kz &= 3 \\ kx - 3y + 5z &= -1 \\ x - 2y + 3z &= -4\end{aligned}$$

Where k is a real constant. The planes do not meet at a unique point.

(a) Find the possible values of k [3 marks]

(b) There are two possible geometric configurations for the given planes. Identify each possible configuration, stating the corresponding value of k . Fully justify your answer. [5 marks]

(c) Given further that the equations of the planes form a consistent system, find the solution of the system of equations. [3 marks]