Topic Z1 Vectors (Pre-TT A) [49] MARKSCHEME

1.

(a)	$\begin{pmatrix}3\\-4\\2\end{pmatrix}\cdot\begin{pmatrix}6\\2\\12\end{pmatrix}=18-8+24$	M1	3.1a
	$d=\dfrac{18-8+24-5}{\sqrt{3^2+4^2+2^2}}$	M1	1.1b
	$=\sqrt{29}$	A1	1.1b
		(3)	
b)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\5\end{pmatrix}=\ldots$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\ldots$	M1	2.1
	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\5\end{pmatrix}=0$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-2\end{pmatrix}=0$ $\therefore\ -\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
c)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-4\\2\end{pmatrix}=-3+12+2$	M1	1.1b
	$\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}\cos\theta=11$ $\Rightarrow\cos\theta=\dfrac{11}{\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}}$	M1	2.1
	So angle between planes $\theta=52°$ *	A1*	2.4
		(3)	
			(8 marks)

2.

<table>
<tr><td>Uses the mathematical model to find the volume by first finding the coordinate of A.
To award this mark must see an attempt to find coords of A, and an attempt at volume of prism</td><td>AO3.4</td><td>M1</td><td rowspan="9">$x = 4t - 4$
$y = 12 - 12t$
$z = 4$

$4t - 4 - 3(12 - 12t) = 0$

$40t - 40 = 0$
$t = 1$
$(0 \quad 0 \quad 4)$
OR
$\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} -4 \\ 12 \\ 4 \end{pmatrix}\right] \times \begin{pmatrix} 4 \\ -12 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 3y+4 \\ y-12 \\ z-4 \end{pmatrix} \times \begin{pmatrix} 4 \\ -12 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
$12(z-4) = 0 \Rightarrow z = 4$
$-12(3y+4) - 4(y-12) = 0$
$\Rightarrow y = 0, x = 0$
A has coordinates $(0,0,4)$

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 9 \\ 3 \\ 0 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -21 \\ 63 \\ 0 \end{pmatrix}$

Area ABC $= \dfrac{21\sqrt{10}}{2}$

$d = 4\sqrt{10}$
Volume =
$V = \dfrac{21\sqrt{10}}{2} \times 4\sqrt{10} = 420\,\text{m}^3$</td></tr>
<tr><td>Selects method involving both equation of plane and equation of line to find coords of A
Either using parametric form or using cross product
Ignore sign errors</td><td>AO3.1a</td><td>M1</td></tr>
<tr><td>Either collects terms together and solves to find value of parameter for ‘their’ equation
Or correctly calculates cross product for ‘their’ vectors</td><td>AO1.1b</td><td>A1F</td></tr>
<tr><td>Deduces the correct coordinates of A</td><td>AO2.2a</td><td>A1</td></tr>
<tr><td>Selects a correct approach to calculate the volume of the prism.</td><td>AO3.1a</td><td>M1</td></tr>
<tr><td>Finds two sides of the triangle ABC in vector form
FT ‘their’ A</td><td>AO1.2</td><td>A1F</td></tr>
<tr><td>Finds area of ABC FT ‘their’ A</td><td>AO1.1b</td><td>A1F</td></tr>
<tr><td>Finds length of prism
FT ‘their’ A</td><td>AO1.1b</td><td>A1F</td></tr>
<tr><td>Gives their answer in context by correctly finding the volume of the roof with correct units.
FT ‘their’ prism</td><td>AO1.1b</td><td>A1F</td></tr>
<tr><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

3.

6	(i)		$l \parallel \begin{pmatrix}2\\3\\5\end{pmatrix}$ $\Pi \perp \begin{pmatrix}4\\-1\\-1\end{pmatrix}$ so $\begin{pmatrix}2\\3\\5\end{pmatrix} \cdot \begin{pmatrix}4\\-1\\-1\end{pmatrix} = 0 \Rightarrow l \parallel \Pi$ (1, –2, 7) on l but $4\times1--2-7=-1\neq8$ so not in Π hence l not in Π	M1 M1 A1 **[3]**	dot product of correct vectors = 0 substitute point on line into Π and calculate d Full argument includes key components	Argument can be about a general point on line
6	(ii)		$(\mathbf{r} =)\begin{pmatrix}1\\-2\\7\end{pmatrix} + \lambda\begin{pmatrix}4\\-1\\-1\end{pmatrix}$ closest point where meets Π $4(1+4\lambda)-(-2-\lambda)-(7-\lambda)=8$ $\Rightarrow \lambda = \frac{1}{2}$ $\Rightarrow \mathbf{r} = \begin{pmatrix}3\\-\frac{5}{2}\\\frac{13}{2}\end{pmatrix}$	B1 M1 A1ft A1 **[4]**	parametric form of (x, y, z) substituted into plane	
6	(iii)		$\mathbf{r} = \begin{pmatrix}3\\-\frac{5}{2}\\\frac{13}{2}\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\5\end{pmatrix}$	B1ft **[1]**	oe	must have "**r** ="

4.

(a)	Uses an appropriate method for ensuring the line lies in the plane	AO3.1a	M1	Let $\lambda = x - p = \frac{y+2}{q} = 3 - z$, then $x = \lambda + p,\ y = q\lambda - 2,\ z = 3 - \lambda$ sub into equation of plane $(\lambda+p)-(q\lambda-2)-2(3-\lambda)+3=0$ $\lambda(3-q)+(p-1)=0$ this is true for all λ therefore p = 1 and q = 3
	Obtains equation(s) in p and q	AO1.1a	M1	
	Deduces the values of p and q	AO2.2a	A1	ALT vector equation of line is $\mathbf{r} = \begin{pmatrix}p\\-2\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\q\\-1\end{pmatrix}$ therefore $\begin{pmatrix}p\\-2\\3\end{pmatrix}$ lies on the plane $\begin{pmatrix}p\\-2\\3\end{pmatrix} \cdot \begin{pmatrix}1\\-1\\-2\end{pmatrix} + 3 = 0$ And $\begin{pmatrix}1\\q\\-1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ therefore $\begin{pmatrix}1\\q\\-1\end{pmatrix} \cdot \begin{pmatrix}1\\-1\\-2\end{pmatrix} = 0$ $\Rightarrow q = 3$ and $p = 1$

(b)	States that to have a solution the coefficient of λ cannot be 0 OR dot product must ≠ 0	AO2.4	R1	$\begin{pmatrix}1\\q\\-1\end{pmatrix}.\begin{pmatrix}1\\-1\\-2\end{pmatrix}\neq 0 \Rightarrow q\neq 3$
	Deduces the range of values for q	AO2.2a	R1	
	Deduces correct range of values for p	AO2.2a	R1	p can take any value
(c)(i)	Finds the correct scalar product of the normal to the plane and the direction vector	AO1.1b	B1	$\mathbf{n}=\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ $\mathbf{d}=\begin{pmatrix}1\\q\\-1\end{pmatrix}$ $\mathbf{n.d} = 3-q$
	Correctly deduces the value of $\cos\ \alpha$	AO2.2a	R1	Let α be angle between the line and the normal to the plane $\sin\theta = \frac{1}{\sqrt{6}} \Rightarrow \cos\alpha = \frac{\pm 1}{\sqrt{6}}$
	Forms an equation connecting all relevant parts using $\mathbf{n.d} = \lvert\mathbf{n}\rvert\lvert\mathbf{d}\rvert\cos\theta$	AO3.1a	M1	$q-3=\sqrt{6}\sqrt{q^2+2}\times\left(\frac{\pm 1}{\sqrt{6}}\right)$
	Obtains correct value for q	AO1.1b	A1	$(3-q)^2 = q^2+2$ $\Rightarrow 6q = 7$ giving $q=\frac{7}{6}$
(c)(ii)	Uses ‘their’ expressions for x and y and ‘their’ value for q and the equation of the plane to form an equation to find p	AO3.1a	M1	$x-p=\frac{y+2}{\frac{7}{6}}=3-z$ $z=0\Rightarrow x=p+3, y=1.5$
	Uses $z=0$ to deduce expressions for x and y in terms of p and q	AO2.2a	R1	$\begin{pmatrix}p+3\\1.5\\0\end{pmatrix}.\begin{pmatrix}1\\-1\\-2\end{pmatrix}=-3$ $\Rightarrow p+3-1.5=-3$ $\Rightarrow p=-4.5$
	Obtains the correct value of p CAO	AO1.1b	A1	
	Total		**13**	

5.

(a)	Uses an appropriate method for finding the values of k (for example expanding appropriate determinant)	AO1.1a	M1	$\begin{vmatrix} 1 & -1 & k \\ k & -3 & 5 \\ 1 & -2 & 3 \end{vmatrix} = 0$
	Obtains a quadratic equation in k	AO1.1a	M1	$\begin{vmatrix} -3 & 5 \\ -2 & 3 \end{vmatrix} + \begin{vmatrix} k & 5 \\ 1 & 3 \end{vmatrix} + k\begin{vmatrix} k & -3 \\ 1 & -2 \end{vmatrix} = 0$
	Obtains two correct values for k	AO1.1b	A1	$1 + 3k - 5 + k(-2k + 3) = 0$ $-2k^2 + 6k - 4 = 0$ $k^2 - 3k + 2 = 0$ $(k-2)(k-1) = 0$ $k = 2$ or 1
(b)	Selects an appropriate method to determine the appropriate geometrical configuration and substitutes 'their' first value of k	AO3.1a	M1	when $k = 1$ $x - y + z = 3$ $x - 3y + 5z = -1$ $x - 2y + 3z = -4$
	Eliminates one variable or uses row reduction	AO1.1a	M1	$-2y + 4z = -4$ $y - 2z = 7$
	Obtains a contradiction and makes correct deduction about the geometric configuration (must have correct value for k)	AO2.2a	R1	$y - 2z = 2; \quad y - 2z = 7$ Hence equations are inconsistent and the three planes form a prism
	Substitutes 'their' 2nd value of k into selected method to determine the appropriate geometrical configuration	AO1.1a	M1	when $k = 2$ $x - y + 2z = 3$ $2x - 3y + 5z = -1$ $x - 2y + 3z = -4$
	Obtains a consistent set of equations and makes correct deduction about geometric configuration (must have correct value for k)	AO2.2a	R1	$R_2 - 2R_1: -y + z = -7$ $R_3 - R_1: -y + z = -7$ Hence equations are consistent and the three planes form a sheaf – they meet in line

(c)	Deduces that the planes must meet in a line and hence that $k=2$	AO2.2a	R1	$x-y+2z=3$ $2x-3y+5z=-1$ $x-2y+3z=-4$ $\Rightarrow -y+z=-7$ Let $z=\lambda$ Then $y=\lambda+7$ and $x=3+y-2z$ $=3+\lambda+7-2\lambda$ $=-\lambda+10$
	Selects method to find solution: For example, sets one variable = λ, substitutes and attempts to find other variables in terms of λ	AO1.1a	M1	**ALT** $\begin{pmatrix}1\\-1\\2\end{pmatrix}\times\begin{pmatrix}1\\-2\\3\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}10\\7\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\-1\end{pmatrix}$
	Fully states correct solution CAO	AO1.1b	A1	
	Total		**11**	