Topic Z1 Vectors (Pre-TT B) [45]

1.

Find the acute angle between the line with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and the plane with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$. [7]

(Total 7 marks)

2.

The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\-1\end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix}3\\0\\1\end{pmatrix} + \mu\begin{pmatrix}4\\-1\\-1\end{pmatrix}$$

respectively.

(i) Find the shortest distance between the lines. [5]

(ii) Find a cartesian equation of the plane which contains l_1 and which is parallel to l_2. [2]

(Total 7 marks)

3.

(a) Determine the values of k for which the system of equations

$$x - 3y + 2z = -7$$
$$kx + y - z = -5$$
$$6x - 5y + (k-1)z = 1$$

does not have a unique solution.

(4)

Given that $k = 5$

(b) (i) by finding an appropriate inverse matrix, solve this system of equations,

(ii) interpret the solution geometrically.

(5)

(Total 9 marks)

4.

The position vectors of the points A, B, C, D, G are given by

$\mathbf{a} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = \mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$, $\mathbf{d} = 3\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}$, $\mathbf{g} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

respectively.

(i) The line through A and G meets the plane BCD at M. Write down the vector equation of the line through A and G and hence show that the position vector of M is $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$. [6]

(ii) Find the value of the ratio $AG : AM$. [1]

(iii) Find the position vector of the point P on the line through C and G, such that $\overrightarrow{CP} = \frac{4}{3}\overrightarrow{CG}$. [2]

(iv) Verify that P lies in the plane ABD. [4]

(Total 13 marks)

5.

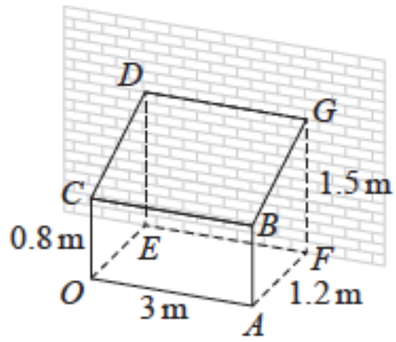

Figure 2

Figure 2 shows a sketch of a shelter against a wall. The shelter consists of two rectangular wooden boards, *OABC* and *BCDG*, which can be modelled as parts of planes. Board *OABC* is vertical and parallel to the wall and the ground may be assumed to be horizontal.
The points *E* and *F* are at the foot of the wall directly below *D* and *G* respectively.

The length *OC* is 0.8 m, the length *OA* is 3 m and the board *OABC* is 1.2 m away from the wall. The points *D* and *G* are 1.5 m above the ground.

To model the shelter, take *O* as the origin, the vector **i** to be 1 m in the direction of $\overrightarrow{OA}$, the vector **j** to be 1 m in the direction of $\overrightarrow{OE}$ and the vector **k** to be 1 m in the direction of $\overrightarrow{OC}$.

(a) Find an equation of the plane *BCDG*, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$

(5)

In order to support the roof of the shelter, one end of a pole is attached to the ground at the centre of the rectangle *OAFE* and the other end to a point on the roof. Modelling the pole as a rod,

(b) find, to the nearest mm, the shortest possible length for the pole.

(3)

(c) State a limitation of the assumption that the boards can be modelled as planes.

(1)

(Total 9 marks)